

Magnetic Resonance Imaging

Ricardo Otazo, PhD

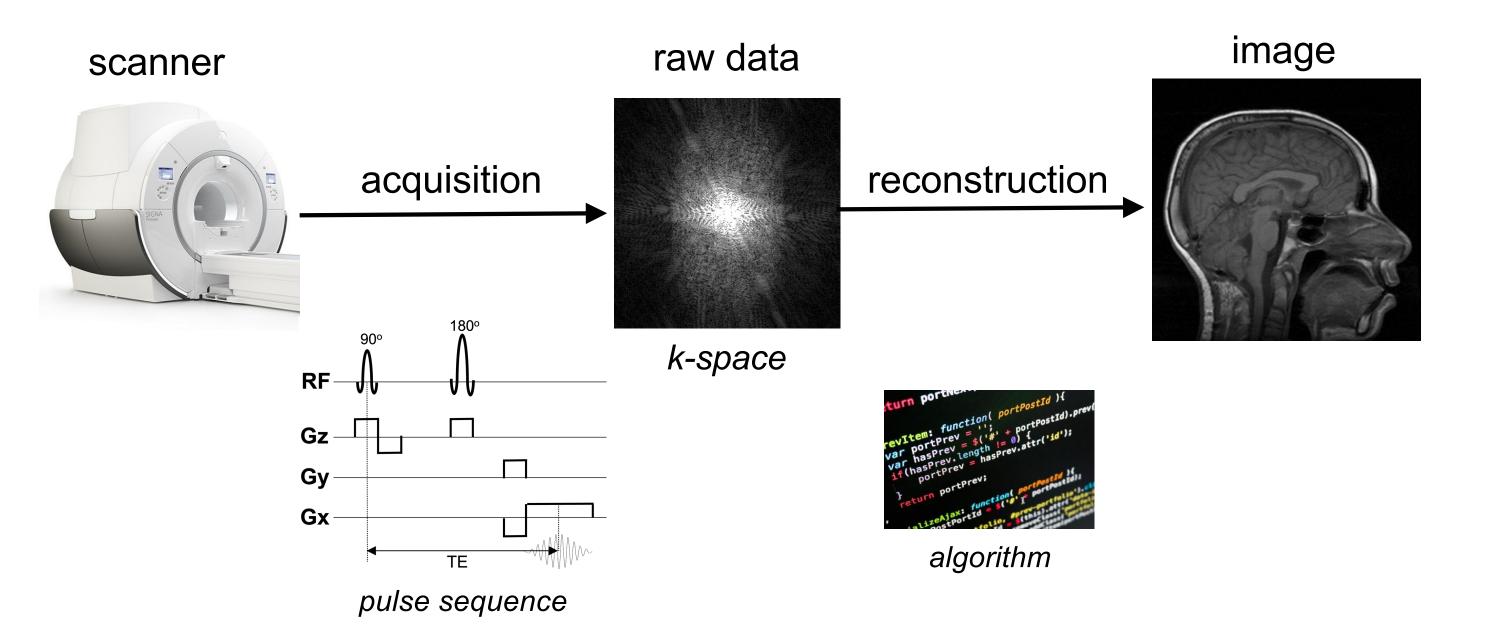
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Agenda

- Imaging with magnetic field gradients
 - k-space
- Pulse sequences
 - Gradient-echo, spin-echo
- Image reconstruction: k-space undersampling
- Parallel imaging, compressed sensing, deep learning
- MR fingerprinting: quantitative imaging
- Experiment in Matlab & Python: accelerated brain MRI

MRI



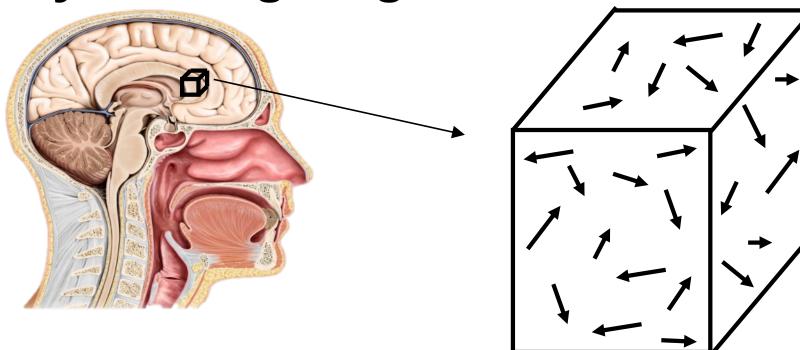
Data acquisition

Gradients, k-space and pulse sequences

Protons and spins

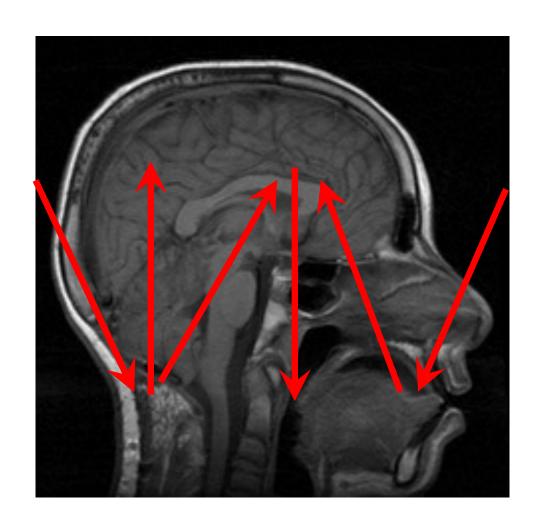
 The nucleus of the hydrogen atom (proton) has a magnetic momentum (spin)

Proton = tiny rotating magnet



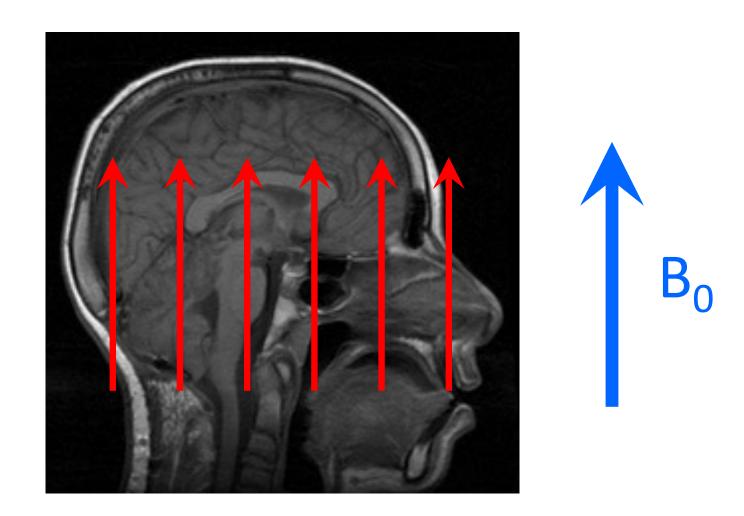
Polarization and magnetization

Spins are randomly oriented (no net magnetization)



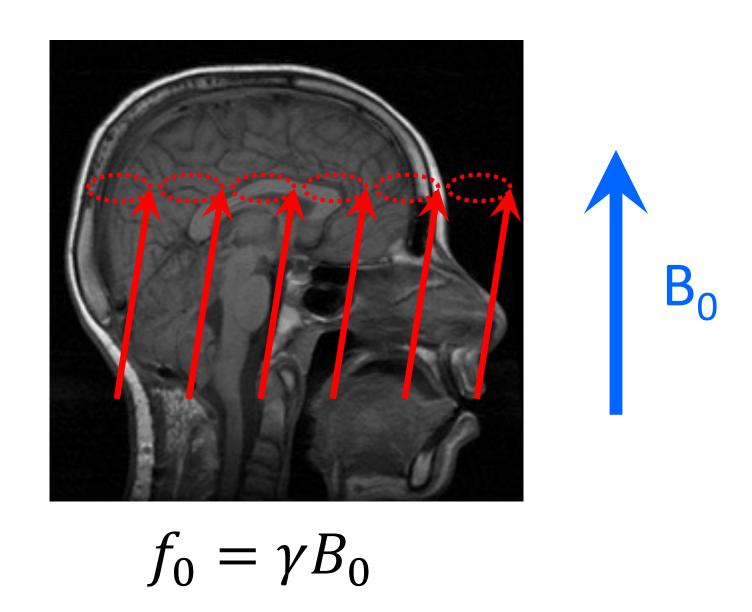
Polarization and magnetization

- Spins are aligned by a strong magnetic field
 - magnetization



Magnetic resonance

Precession proportional to the magnetic field strength



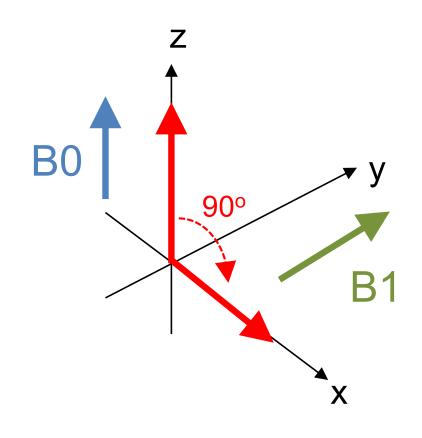
Magnetic resonance

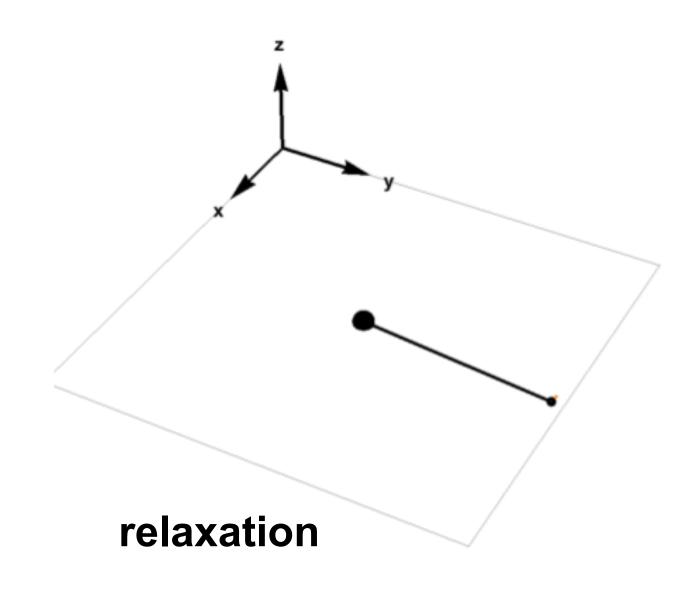
• $\gamma = 42.58~MHz/T$ (gyromagnetic ratio for ¹H)

```
• f_0 = 63.9 \text{ MHz} (1.5\text{T})
= 127.74 MHz (3T)
```

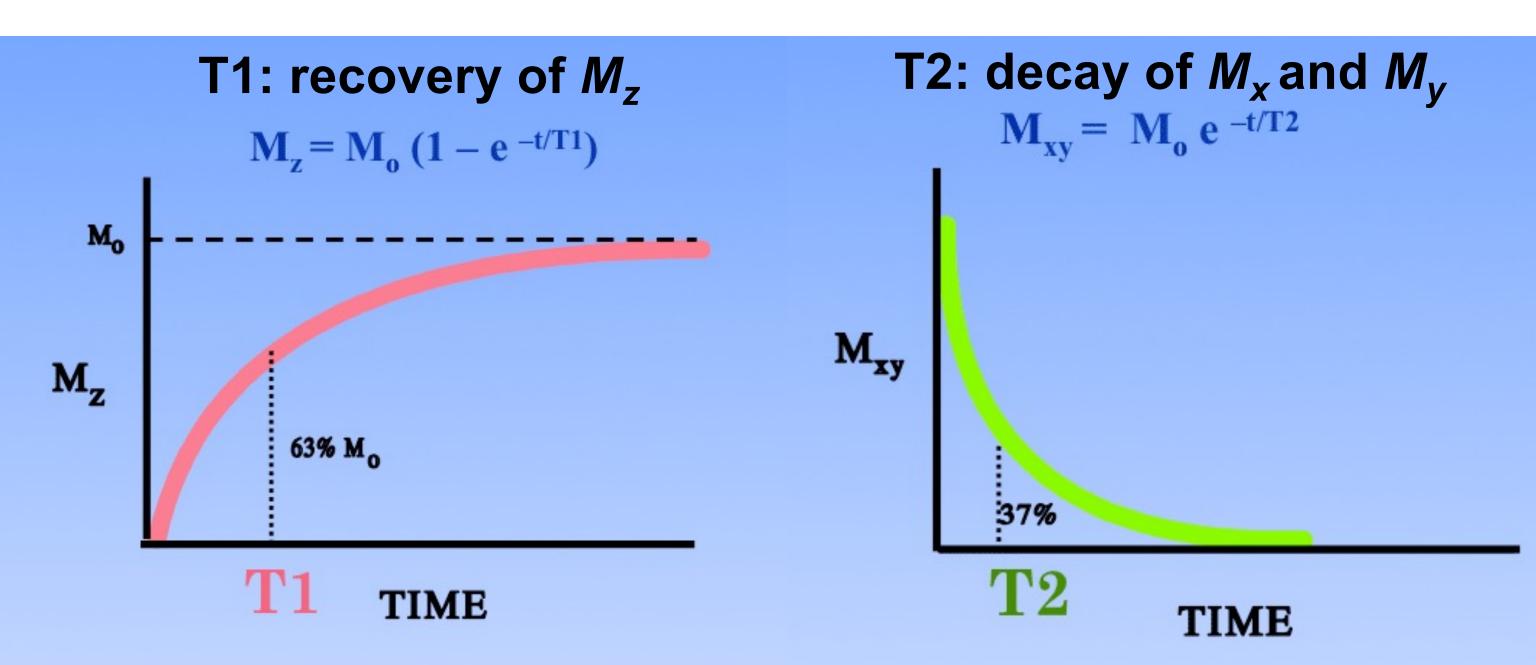
RF excitation (B1)

- Displacing the spins from equilibrium
- Produce a detectable magnetization





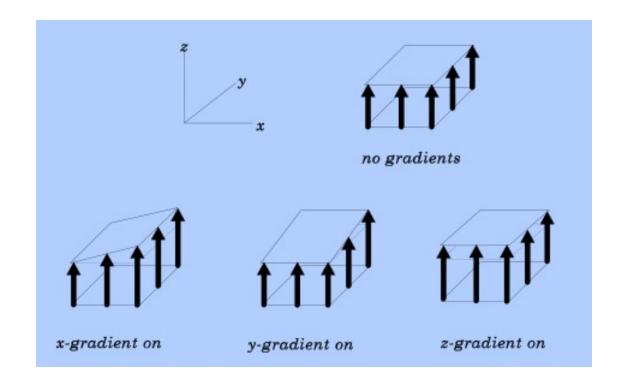
Relaxation (T1, T2)

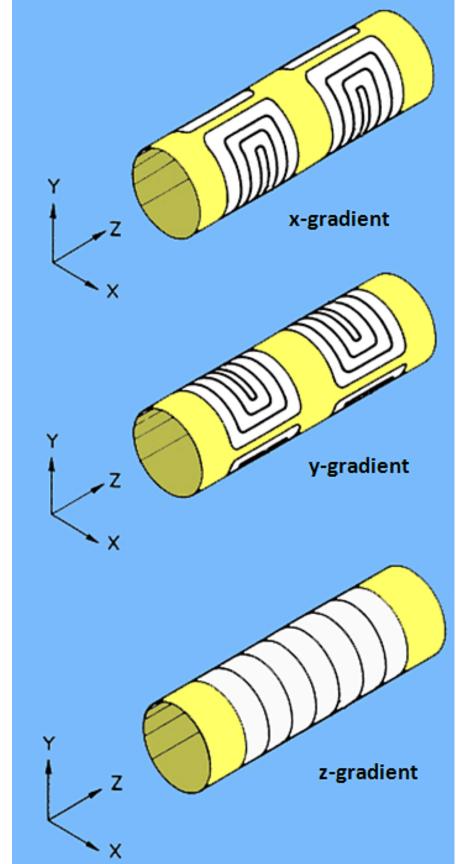


Imaging

- Magnetic field gradients
 - Spatially-varying magnetic fields

$$B(x,y,z) = B0 + G_{\chi}x + G_{y}y + G_{z}z$$





Signal equation

 After B0 demodulation, only magnetic field gradients contribute to the MR signal:

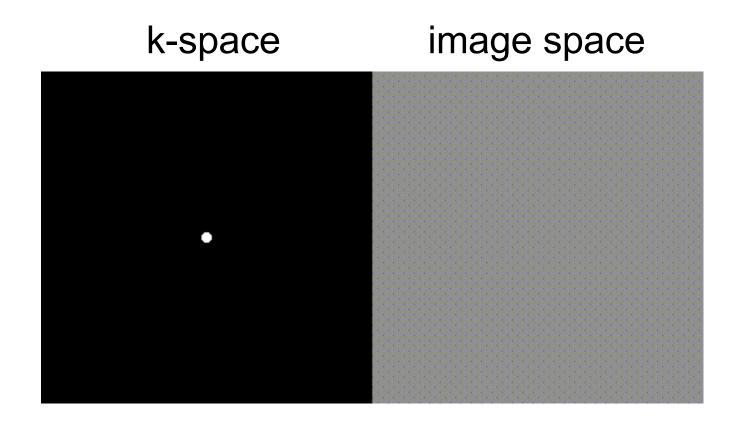
$$S(t) = \int_{r} M(r)e^{i2\pi\gamma \int_{0}^{t} G(\tau)d\tau \cdot r} dr$$

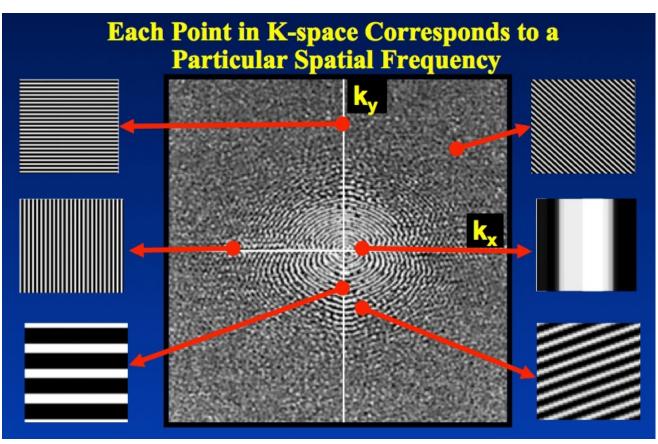
- Defining:
$$k(t) = \gamma \int_0^t G(\tau) d\tau$$
 (k-space)

$$s(t) = \int_{r} M(r)e^{i2\pi k(t)\cdot r}dr$$
 Fourier transform of $M(r)$

k-space

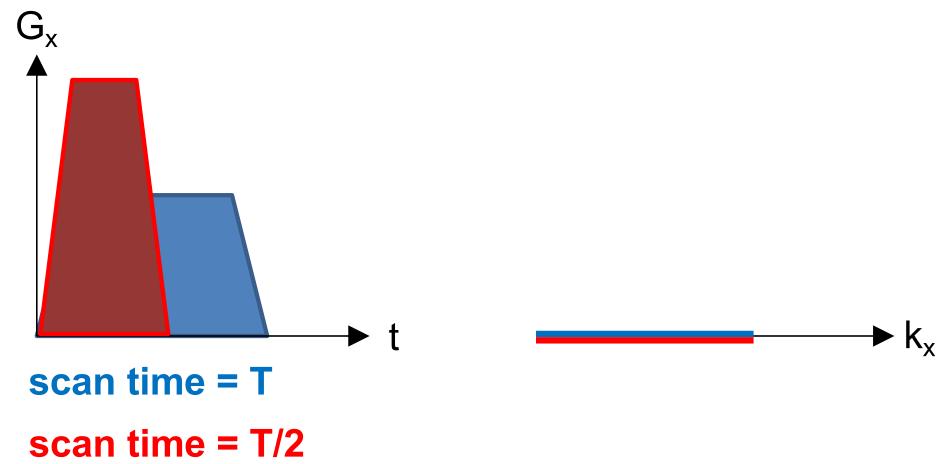
Space of spatial frequencies





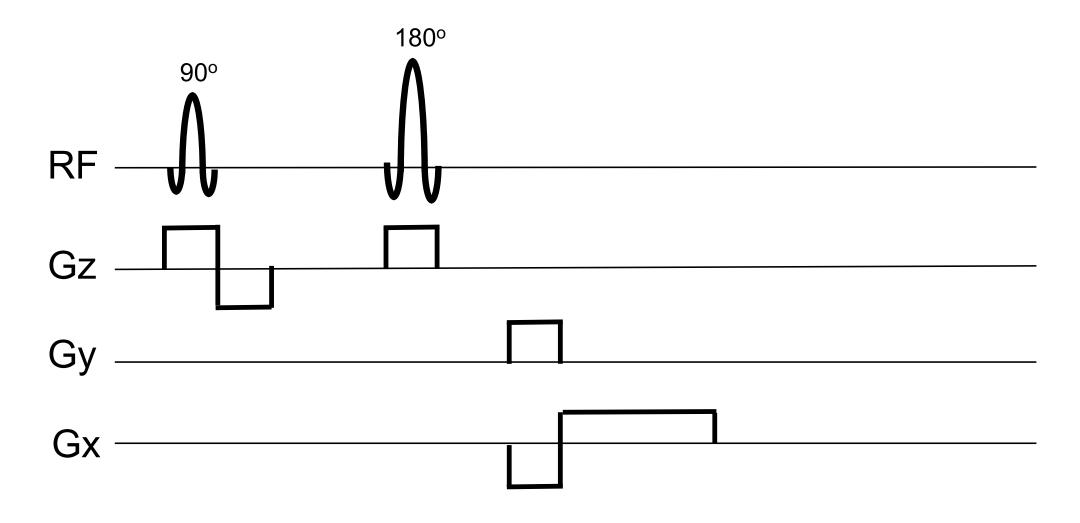
k-space

• Gradient moment
$$k(t) = \gamma \int_0^t G(\tau) d\tau$$



Pulse sequence

 Diagram of the temporal waveforms used for the RF (B1) and gradient (Gx, Gy and Gz) pulses



Pulse sequence

- Program to acquire MRI data
 - Write source code
 - Compile source code
 - Run compiled code on the scanner

ksgre: GRE sequence (2D/3D)

Data Structures

```
struct KSGRE_SEQUENCE
```

Macros

```
#define KSGRE_MINHNOVER 16
#define KSGRE_DEFAULT_SSI_TIME 200 /* which may allow us to use the same SSI for other sequence modules too */
#define KSGRE_DEFAULT_SSI_TIME_SSFP 100
#define KSGRE_INIT_SEQUENCE
```

Functions

```
STATUS cvinit (void)
STATUS cveval (void)
STATUS my_cveval (void)
STATUS cvcheck (void)
STATUS predownload (void)
STATUS pulsegen (void)
STATUS mps2 (void)
STATUS aps2 (void)
STATUS scan (void)
        abstract ("GRE [KSFoundation]")
        psdname ("ksgre")
STATUS ksgre_pg (int start_time)
    int ksgre_scan_coreslice (const SCAN_INFO *slice_pos, int dabslice, int shot, int exc)
    int ksgre_scan_coreslice_nargs (const SCAN_INFO *slice_pos, int dabslice, int nargs, void **args)
    int ksgre_scan_sliceloop (int slperpass, int passindx, int shot, int exc)
    int ksgre_eval_ssitime ()
   void ksgre_init_imagingoptions (void)
STATUS ksgre_init_UI (void)
STATUS ksgre_eval_UI()
STATUS ksgre_eval_setupobjects()
STATUS ksgre_eval_TErange ()
STATUS ksgre_eval_inversion (KS_SEQ_COLLECTION *seqcollection)
STATUS ksgre_eval_tr (KS_SEQ_COLLECTION *seqcollection)
STATUS ksgre_eval_scantime()
STATUS ksgre_check()
STATUS ksgre_predownload_plot (KS_SEQ_COLLECTION *seqcollection)
STATUS ksgre_predownload_setrecon()
  float ksgre_scan_phase (int counter)
STATUS ksgre_scan_init (void)
STATUS ksgre_scan_prescanloop (int nloops, int dda)
```

Variables

```
KS_SEQ_COLLECTION seqcollection

float ksgre_excthickness = 0

float ksgre_gscalerfexc = 0.9

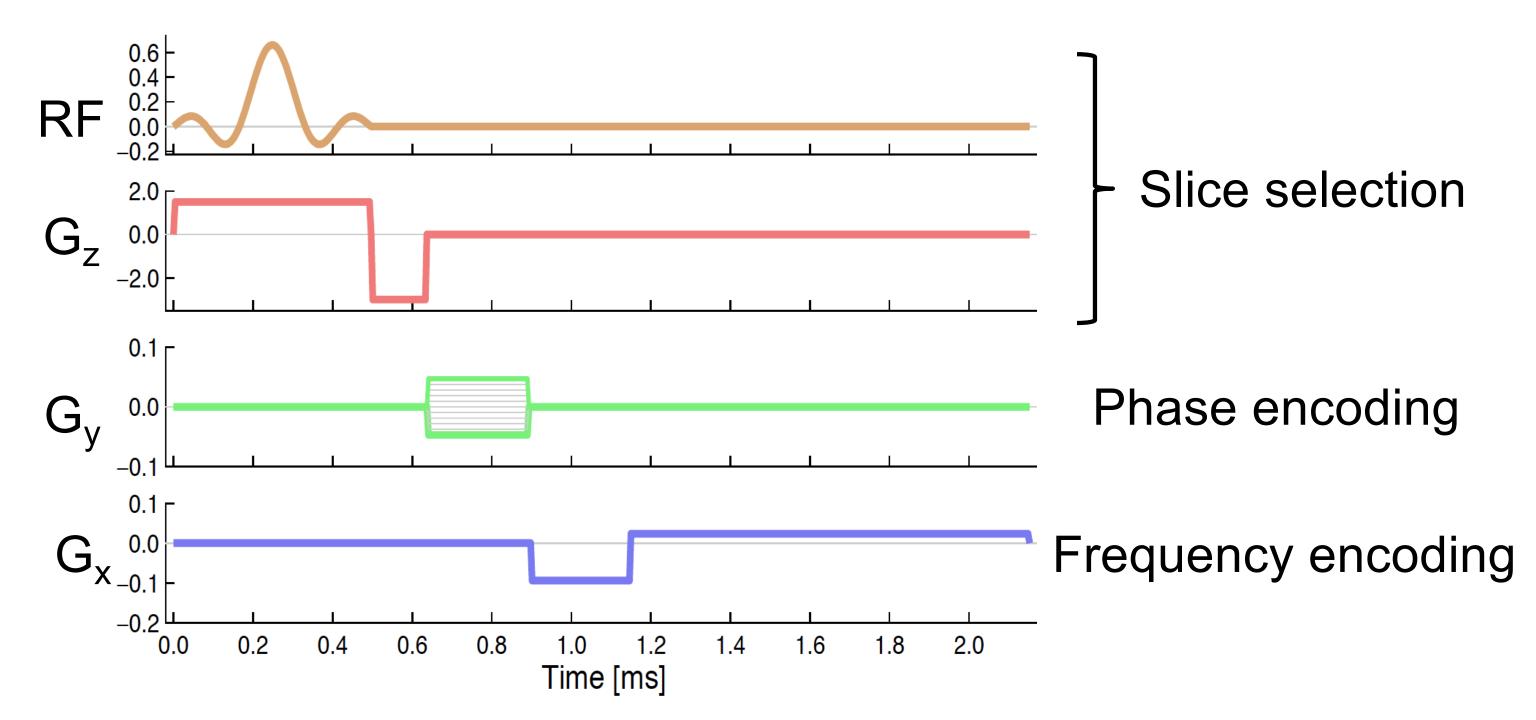
int ksgre_slicecheck = 0 with {0, 1, 0, VIS, "move readout to z axis for slice thickness test",}

float ksgre_slicecheck = 1500 0 with {0, 0, 10000 0, 1500 0, VIS, "ksgre_speiler gradient area"}
```

Imaging techniques

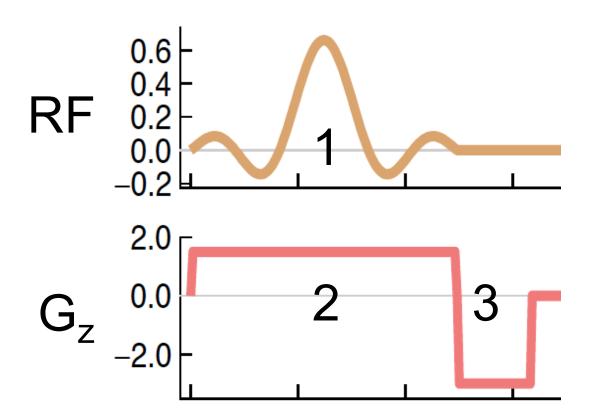
- Slice-selective excitation (z)
- Phase encoding (y)
- Frequency encoding (x)

2D Fourier encoding (Cartesian k-space encoding)



Slice selection

- a.k.a. slice-selective excitation
- Combination of a RF pulse and a gradient



1 – RF pulse

2 – Slice selection gradient

$$\Delta f = \gamma G_z \Delta z$$

3 – Slice refocusing gradient

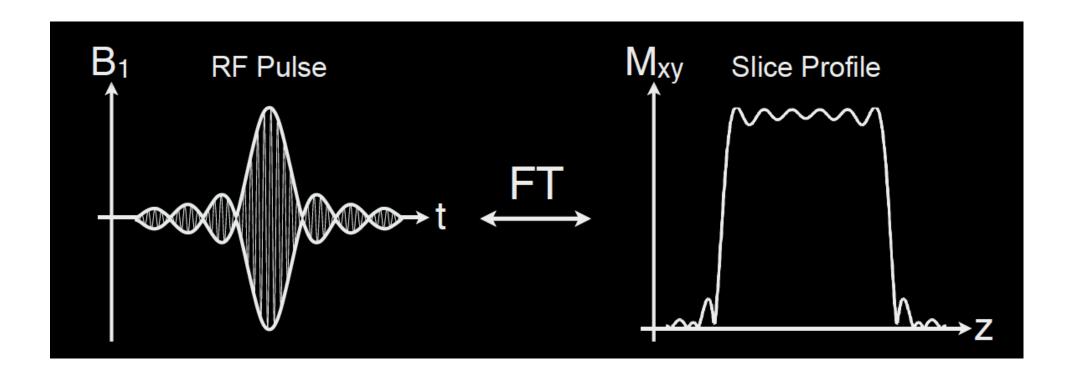
More information at https://mriquestions.com/slice-selective-excitation.html

RF pulses

Center frequency and bandwidth matched to the slice frequencies

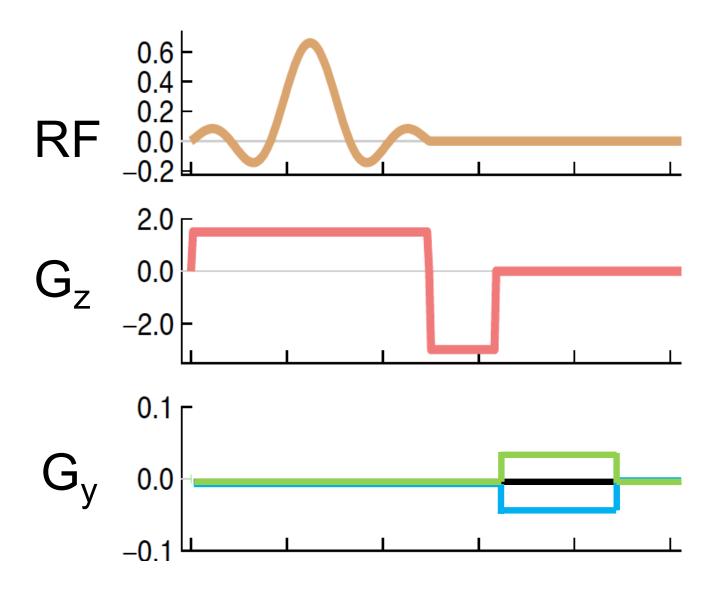
$$\Delta f = \gamma G_z \Delta z$$

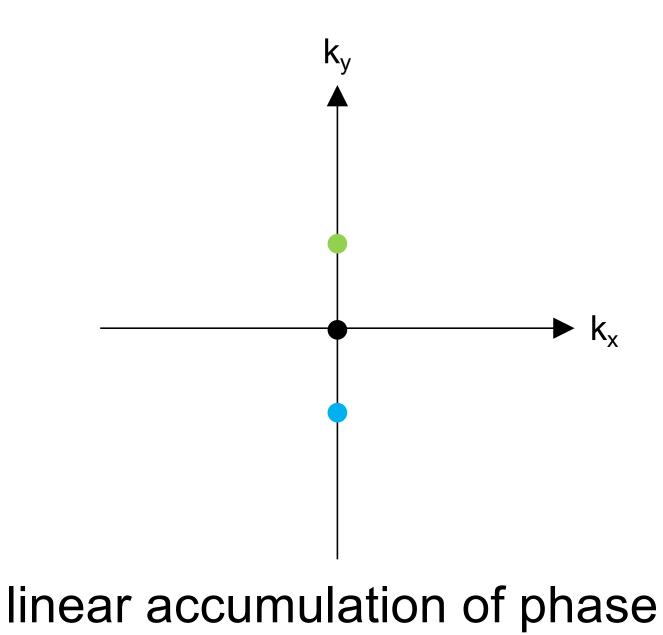
- Slice profile = FT of the envelope
 - Ideal RF pulse: sinc function, square slice profile, too long in practice



Phase encoding

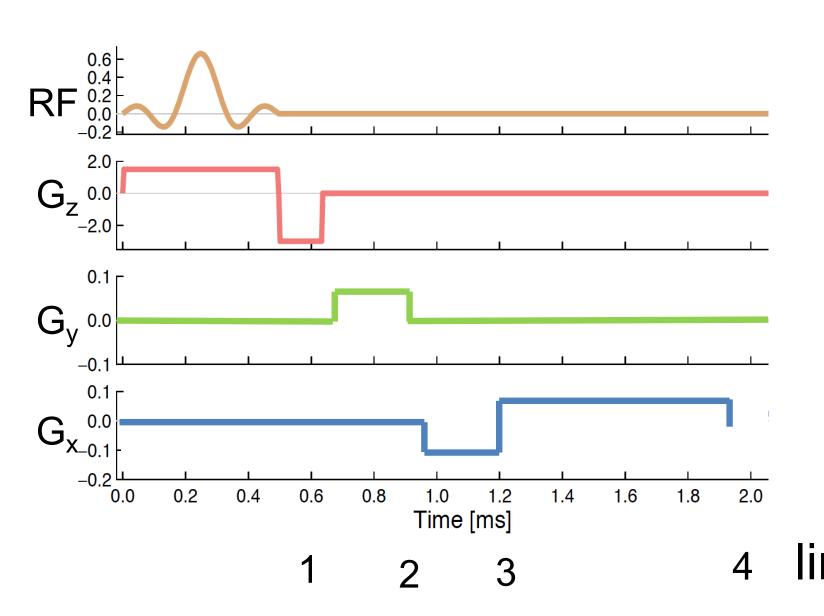
Select one k_y position

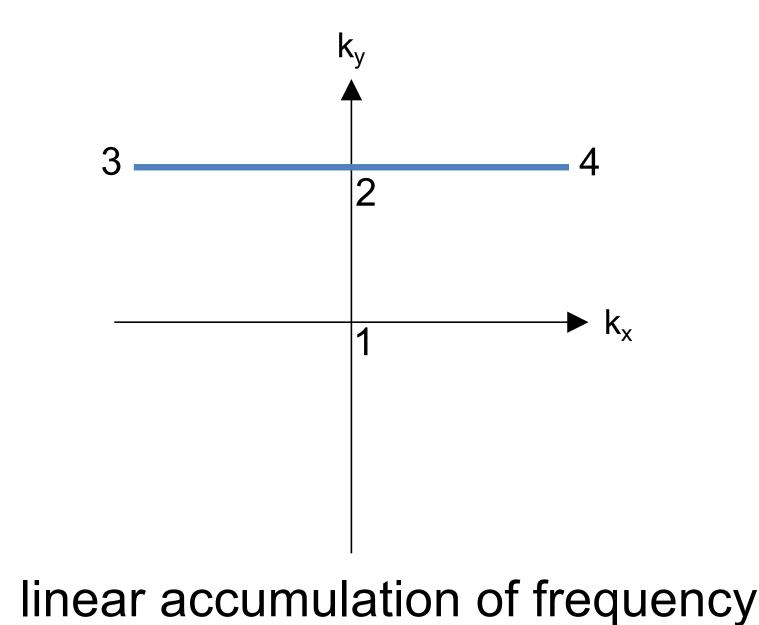




Frequency encoding

Traverse a k_x line

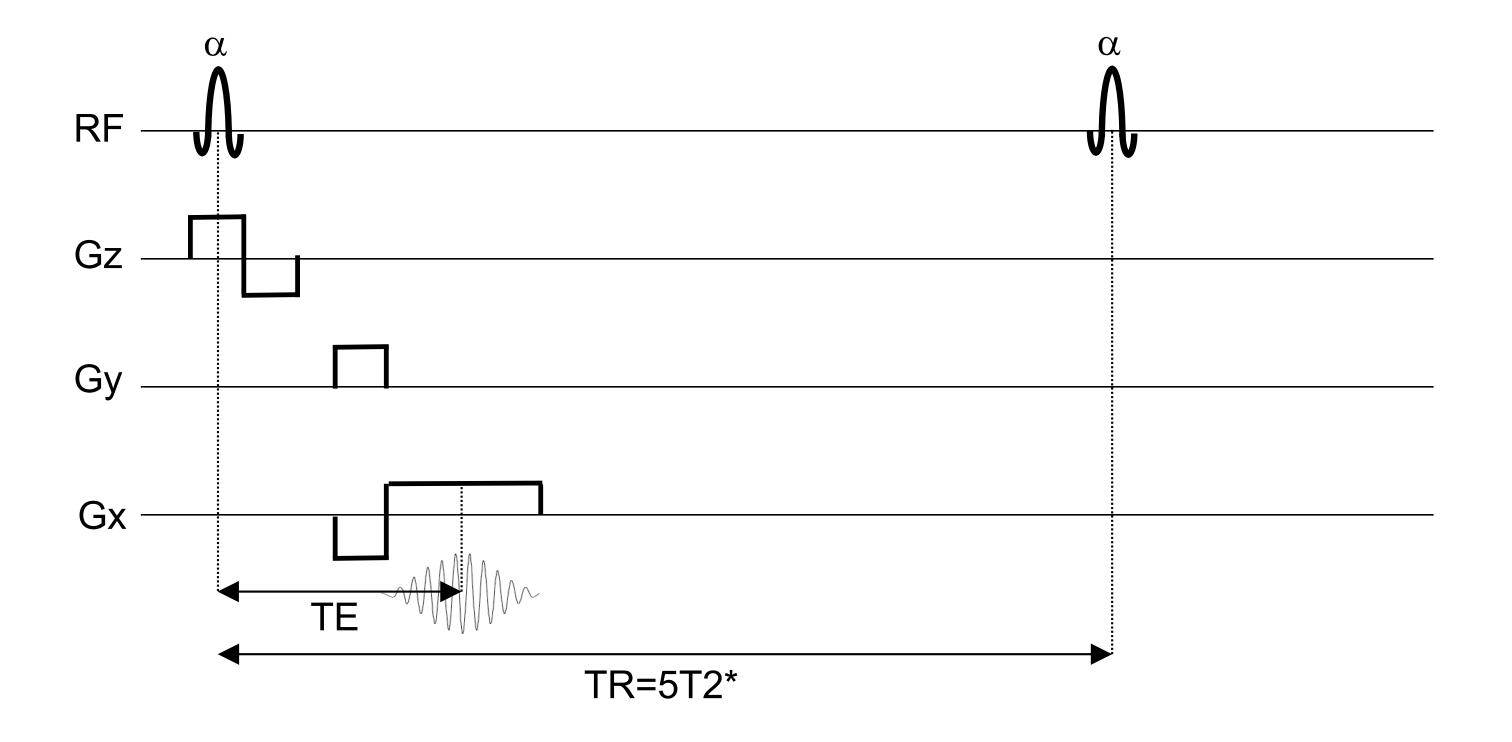




Gradient echo sequences

- Gradient echo sequences use a combination of one RF pulse and gradients to produce an echo
 - No refocusing 180° RF pulse
- Each excitation samples a line in k-space
- The flip angle can be lower than 90°, which enables to reduce the repetition time (TR) and thus scan time
 - Higher flip angles increase T1 weighting
 - Lower flip angles increase T2* weighting

Gradient echo sequence

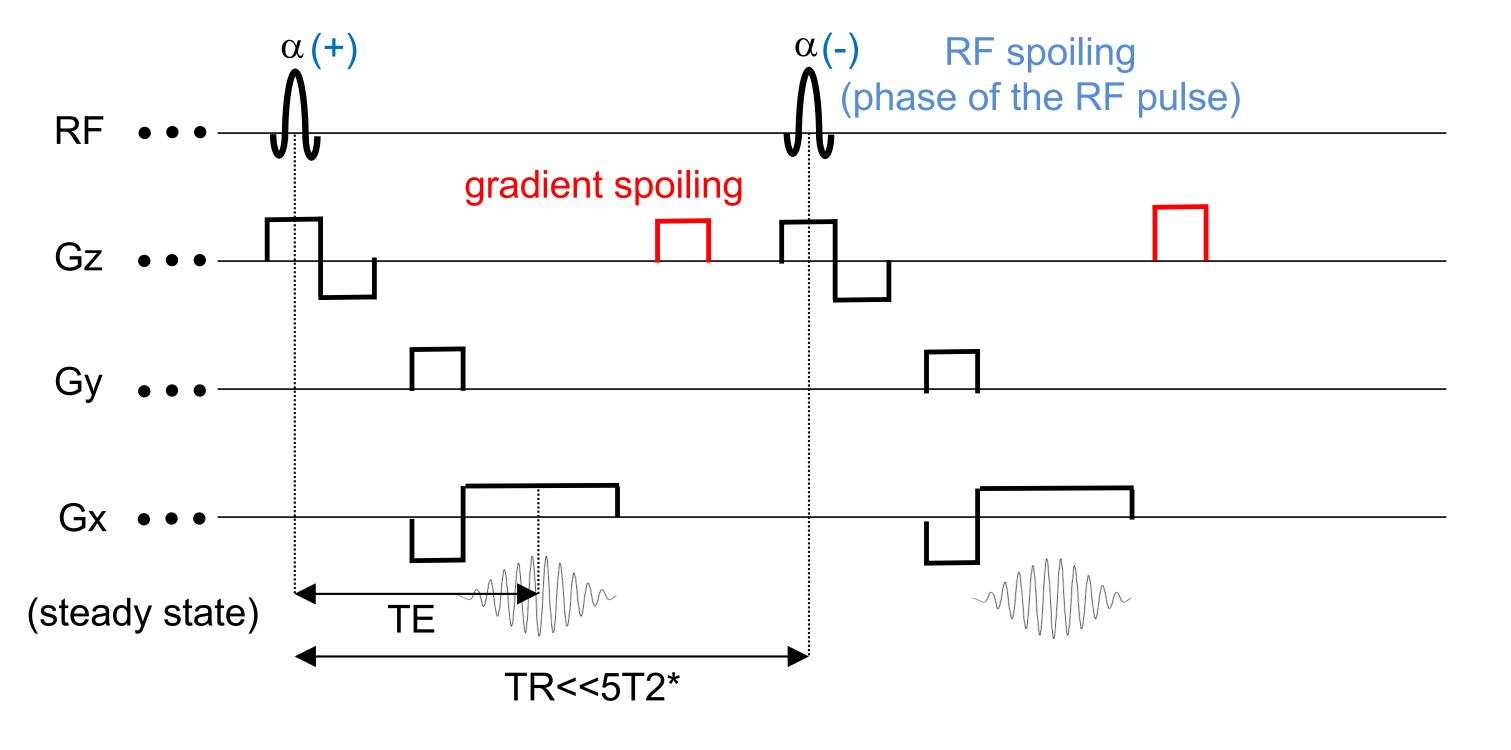


Spoiling

- **Spoiling** refers to the elimination of the transverse component of the magnetization before the next RF pulse in a gradient echo sequence
- The implicit spoiling mechanism is to have a TR of 5T2*, which limits the applicability of gradient echo sequences
- There are two main spoiling mechanisms to have a short TR:
 - Gradient spoiling: a slice-selective gradient with a different amplitude
 - RF spoiling: change the phase of the RF pulse

More information at https://mriquestions.com/spoiling---what-and-how.html

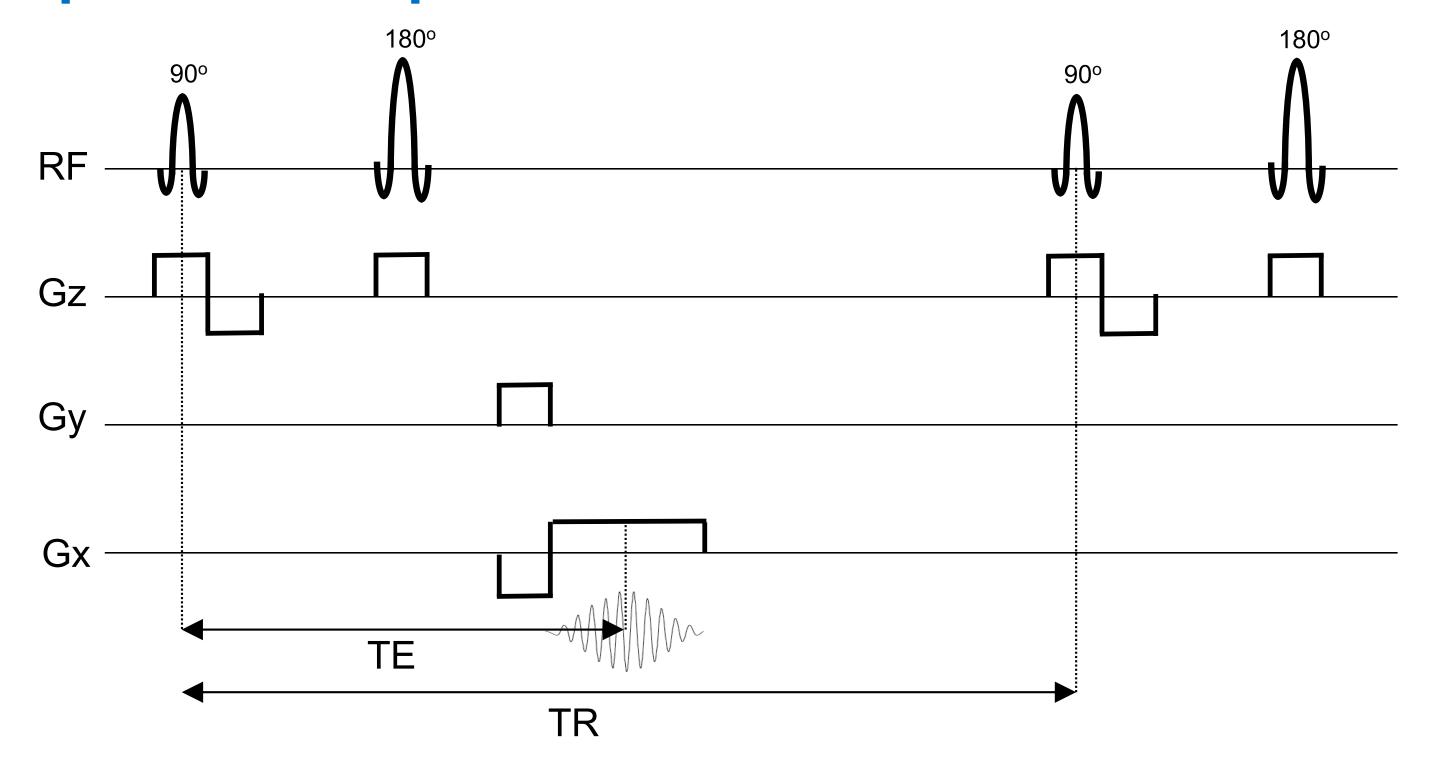
Spoiled gradient echo sequence



Spin echo sequences

- The spin echo sequence uses a 90° excitation pulse, 180° refocusing pulse at TE/2 and signal reading centered at TE.
 This series is repeated for each TR. With each repetition, a k-space line is filled, thanks to a different phase encoding.
- The 180° refocusing pulse compensates for the magnetic field heterogeneities to obtain an echo that is weighted in T2 and not in T2*.

Spin echo sequence



Contrast

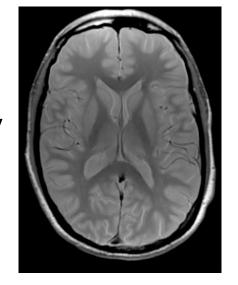
$$S = S_0 \left(1 - e^{-\frac{TR}{T_1}} \right) e^{-\frac{TE}{T_2}}$$

TR

Short TR, short TE

- Incomplete recovery
- Minimal signal decay

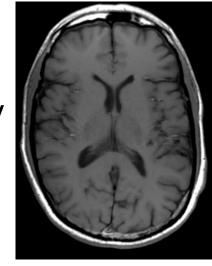
T1 weighting



Long TR, short TE

- Full recovery
- Minimal signal decay

Proton density weighting

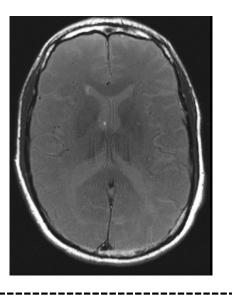


S=0

Short TR, long TE

- Incomplete recovery
 - Signal decay

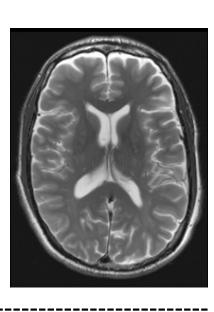
Mixed weighting (not used)



Long TR, long TE

- Full recovery
- Signal decay

T2 weighting



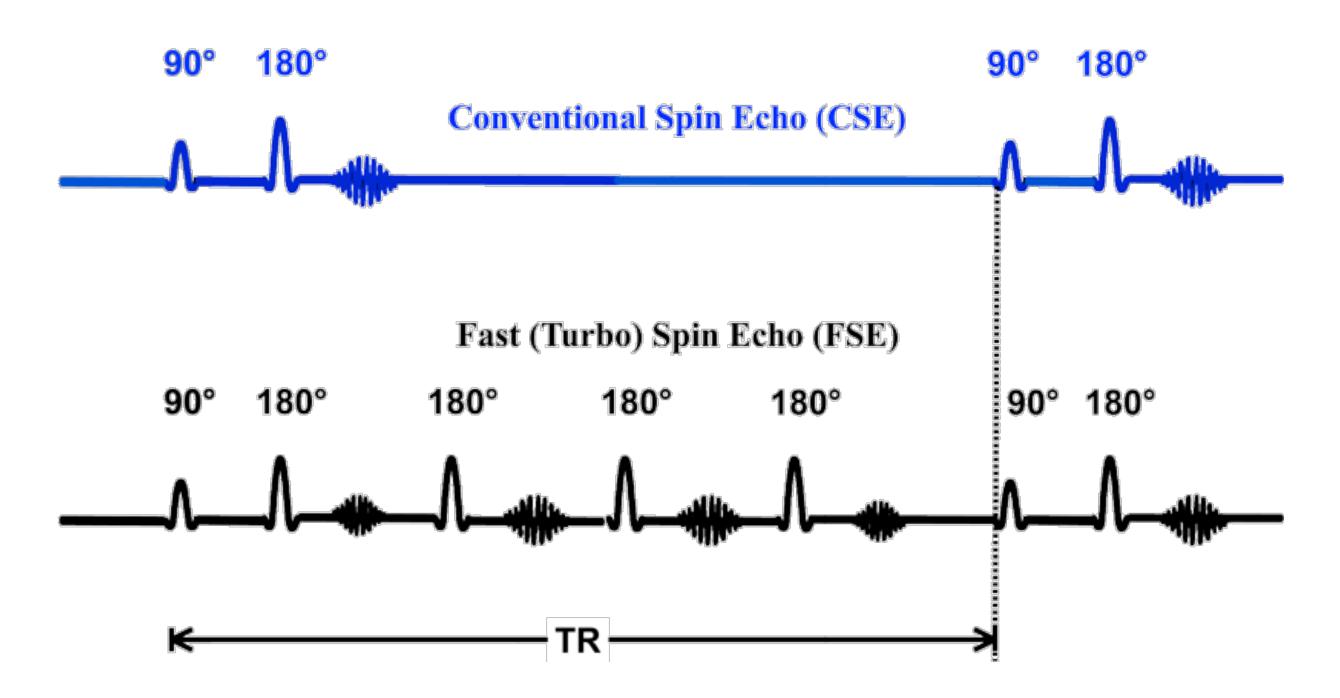
ΤE

S=0 (very long TE)

Turbo Spin Echo (TSE) or Fast Spin Echo (FSE)

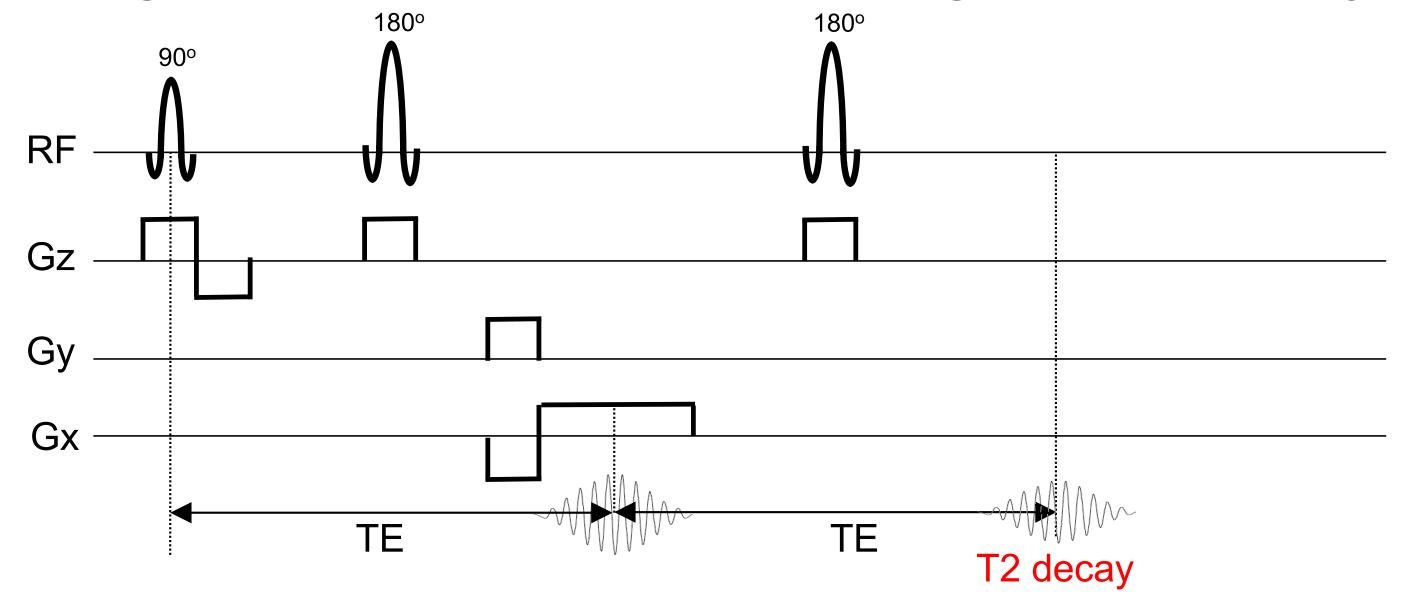
- Fast spin echo (FSE) imaging, also known as Turbo spin echo (TSE) imaging, are commercial implementations of the RARE (Rapid Acquisition with Relaxation Enhancement) technique originally described by Hennig et al in 1986.
- FSE uses one 90° pulse followed by a series of 180° pulses. Phase encoding is applied for each 180° pulse such that multiple k-space lines can be acquired for each TR (echo train), which reduces the total acquisition time
- FSE is one of the workhorse sequences in modern MRI

Turbo Spin Echo (TSE) or Fast Spin Echo (FSE)



Turbo Spin Echo (TSE) or Fast Spin Echo (FSE)

- The signal for each echo in the train is modulated by T2
- Long echo trains can result in blurring due to T2 decay



Test your knowledge: True or False

 Gradient echo sequences do not require an RF pulse to form an echo

FALSE

Every MRI sequence, including gradient echo sequences, require a RF pulse to flip the spins and generate a signal.

Test your knowledge: True or False

 Fat (short T1) appears bright on T1-weighted images and dark on T2-weighted images

TRUE

Fat has a short T1, which allows it to recover signal quickly, appearing bright on T1-weighted images, and has a short T2, which causes it to lose signal quickly, appearing darker on T2-weighted images compared to water.

Test your knowledge: True or False

Spin echo has a shorter echo time (TE) than gradient echo

FALSE

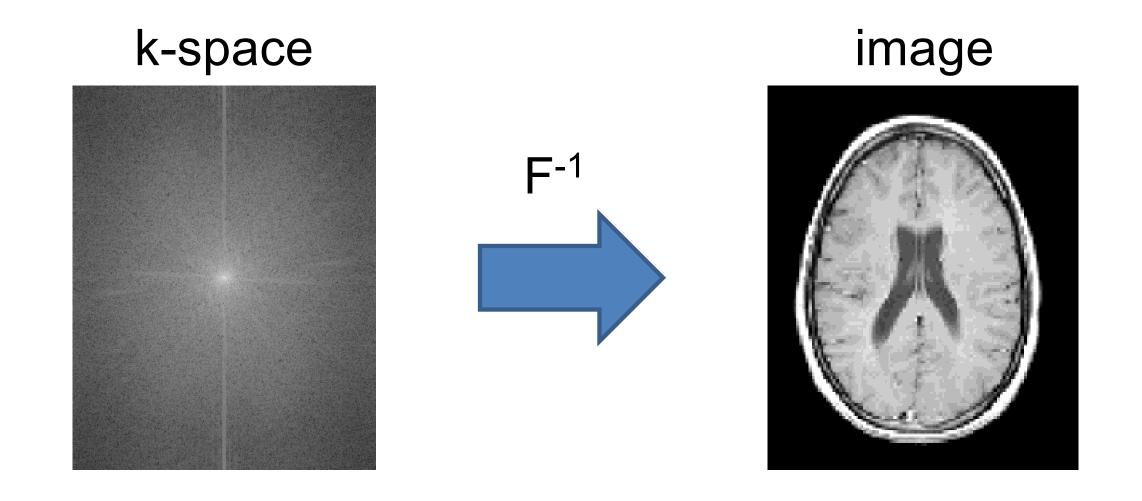
Spin echo sequences require a longer TE because they must wait for the 180° pulse to refocus the spins, and this process takes more time compared to the use of a refocusing gradient.

Image reconstruction

Fourier, parallel imaging, compressed sensing, deep learning

Image reconstruction

- Turn acquired k-space data into an image
- Inverse Fourier transform

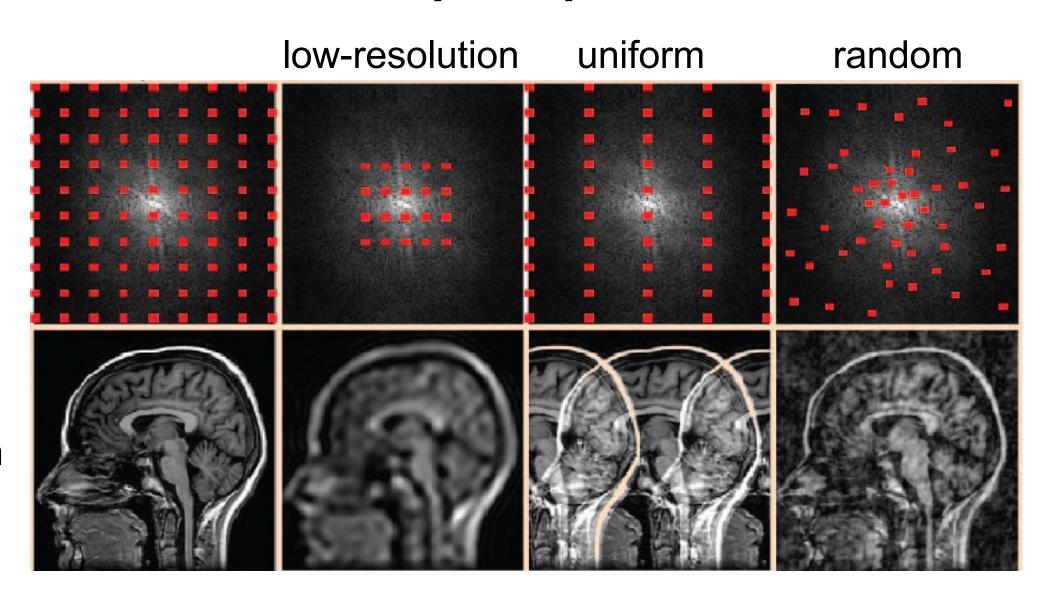


k-space undersampling = acceleration

Reduce the number of k-space points

k-space sampling pattern

Fourier reconstruction



k-space undersampling = acceleration

Reconstruction: fill-in k-space or unalias images

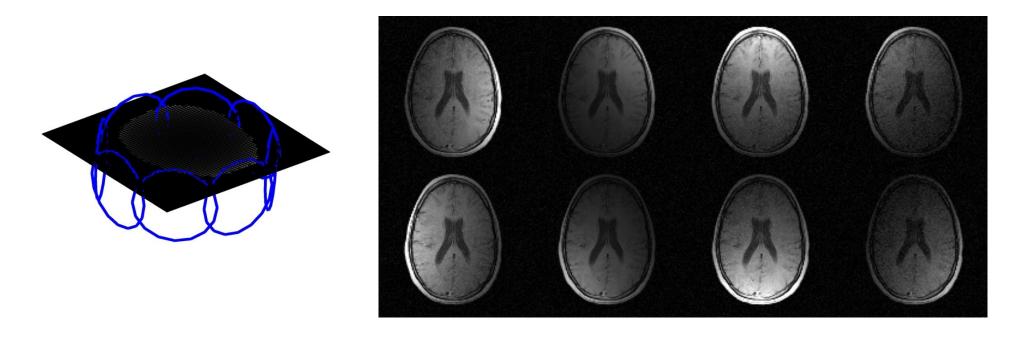
Parallel imaging: Multiple coils

Compressed sensing: Image compressibility

Deep learning: Neural network learning

Parallel imaging

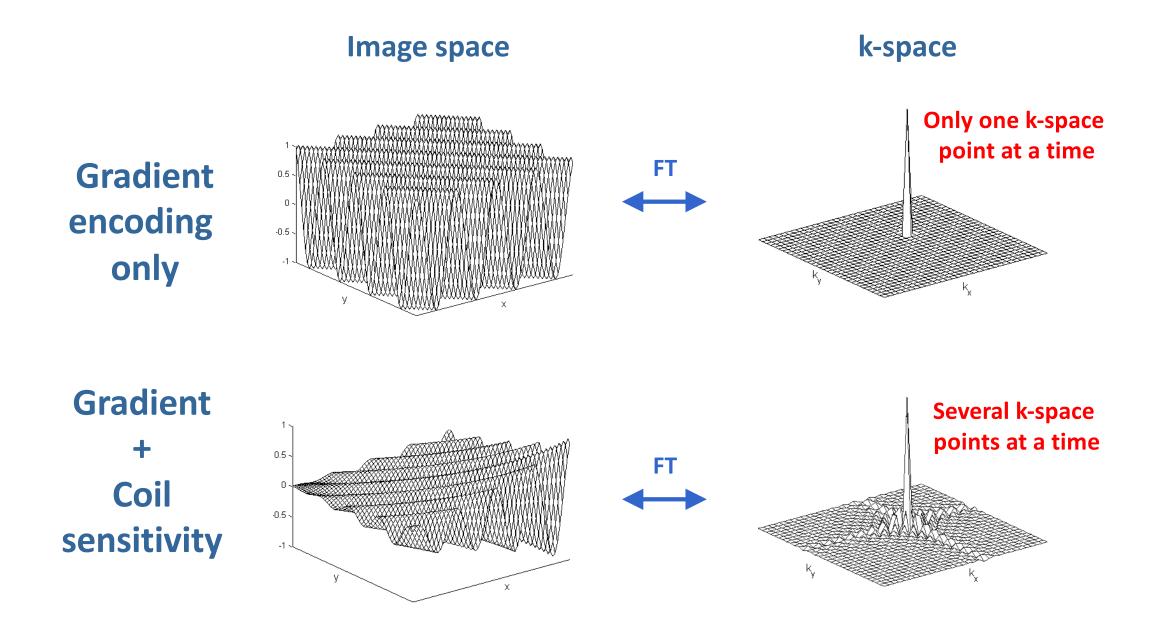
Multiple receiver coils with different spatial sensitivities



$$y_i(r) = m(r)c_i(r)$$

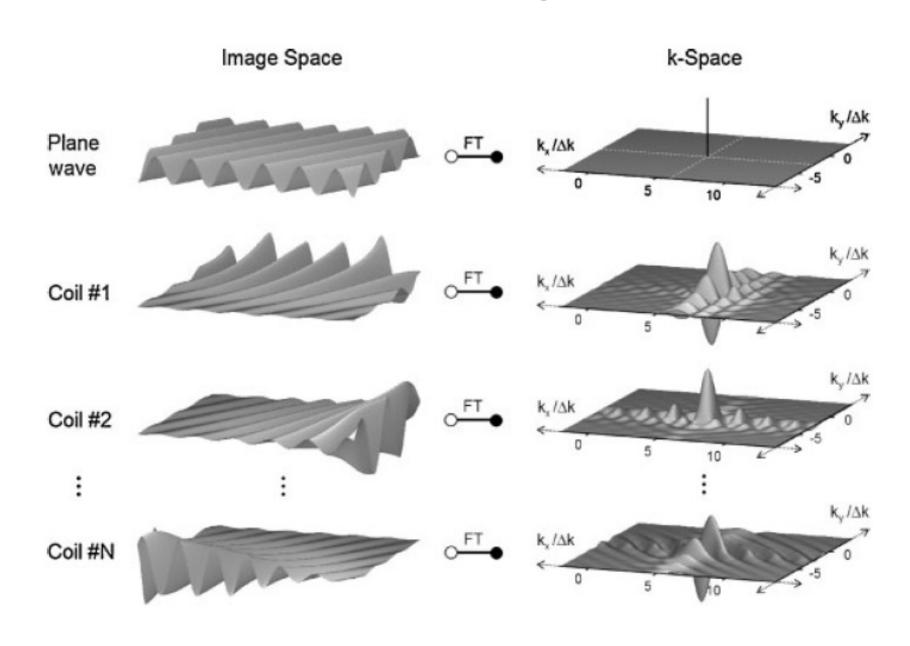
- Uniform k-space undersampling
- Matrix inverse (linear)

Gradient + coil-sensitivity encoding



Gradient + coil-sensitivity encoding

Effective k-space oversampling



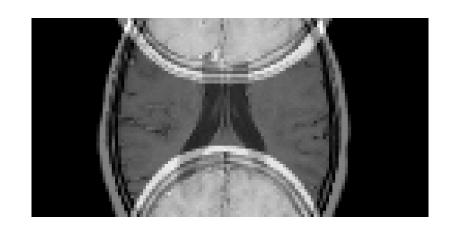
Parallel imaging reconstruction

- Image domain
 - Unfold aliased images
 - SENSE

- k-space
 - Estimate missing k-space points
 - Interpolation
 - SMASH, GRAPPA

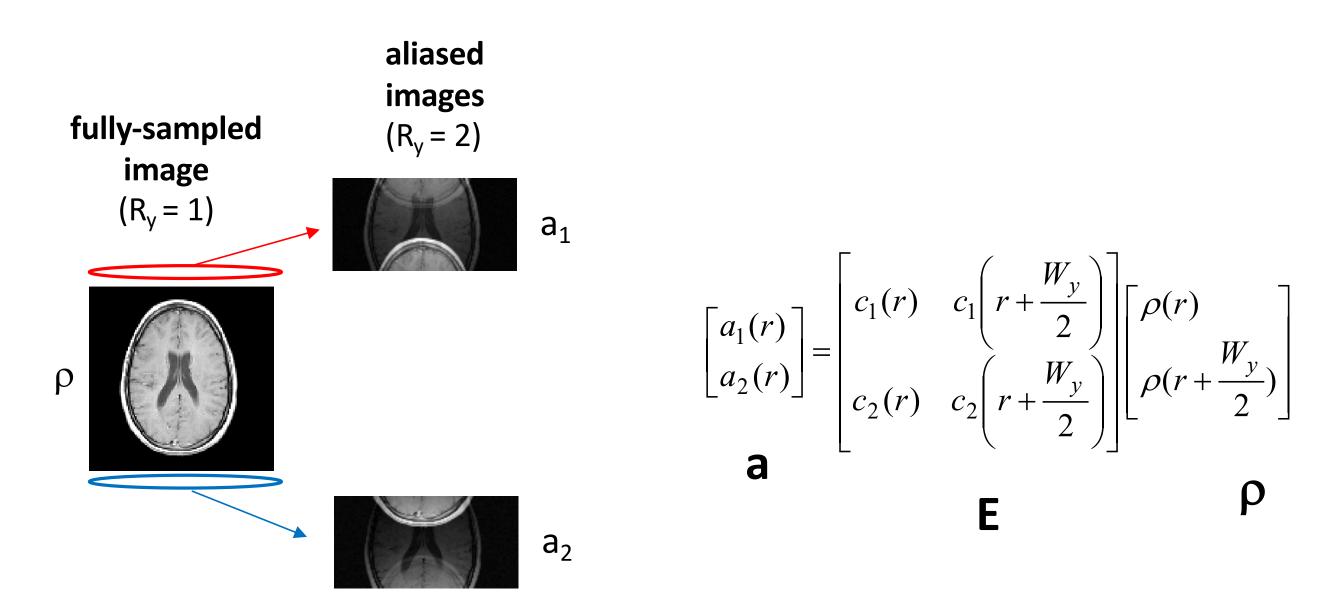
SENSE reconstruction

- Image domain
 - FOV is reduced by R
 - Each point in the aliased image is a combination of R points from the fully-sampled image



SENSE reconstruction

Encoding equation (linear model: matrix)



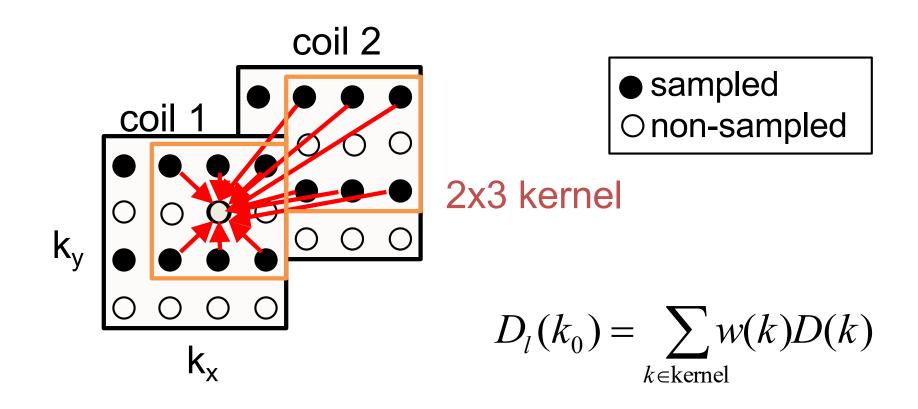
SENSE reconstruction

- Inverse of the encoding equation
 - Undersampling factor < number of coils
 - Pseudoinverse of E

$$\hat{\mathbf{\rho}} = \left(\mathbf{E}^H \mathbf{E}\right)^{-1} \mathbf{E}^H \mathbf{a}$$

GRAPPA

- Coil-by-coil k-space reconstruction
- Linear combination of k-space neighbors from all coils



Griswold MA et al. Magn Reson Med 2002; 47: 1202-10

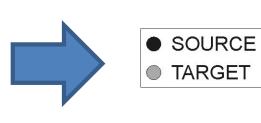
GRAPPA

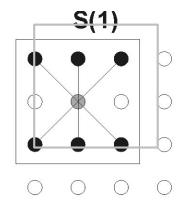
Reconstruction weights (GRAPPA kernel)

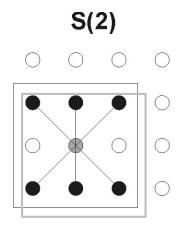
ACS: 4x4 matrix

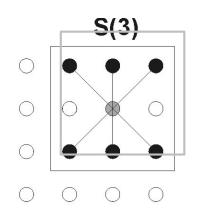
kernel size: 2x3

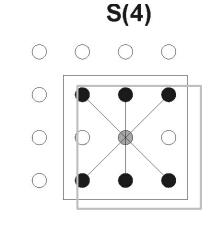
 $R_y=2$











Calibration model: T = Sw

S: source matrix $(N_b \times K_{size}N_c)$

T: target matrix $(N_b \times N_c)$

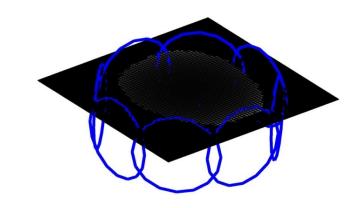
Invert to get the weights: $\mathbf{W} = (\mathbf{S}^{\mathsf{H}}\mathbf{S})^{-1}\mathbf{S}^{\mathsf{H}}\mathbf{T}$ $(N_{\mathsf{b}} \times K_{\mathsf{size}}N_{\mathsf{c}})$

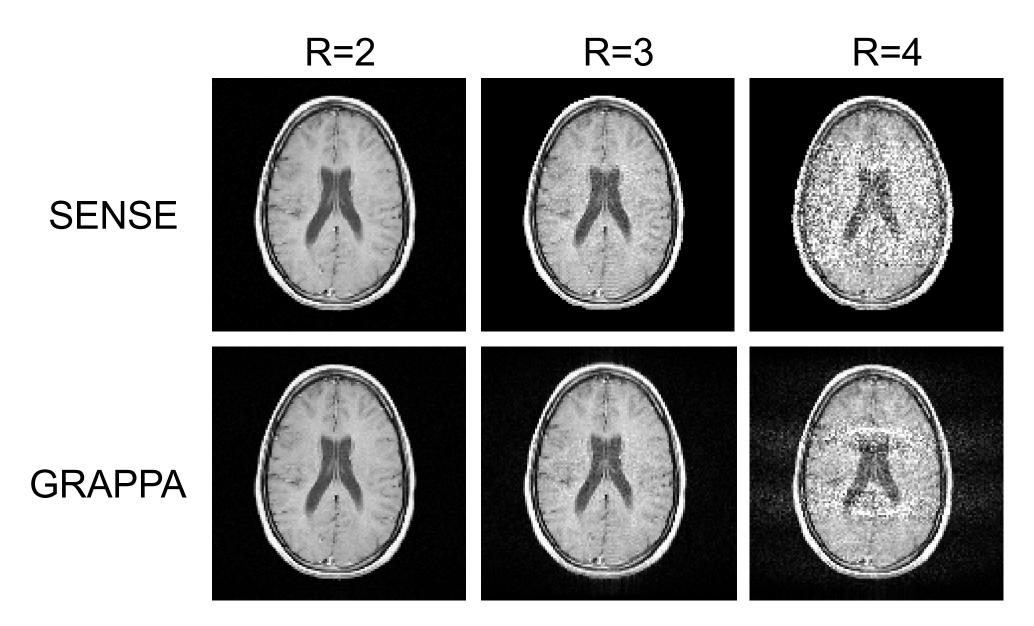
GRAPPA algorithm

- Compute GRAPPA weights from calibration data
- Compute missing k-space data (coil-by-coil reconstruction)
- Inverse FFT to each coil individual coil images and coil combination

zero-pad at the border calibration region

Reconstruction examples



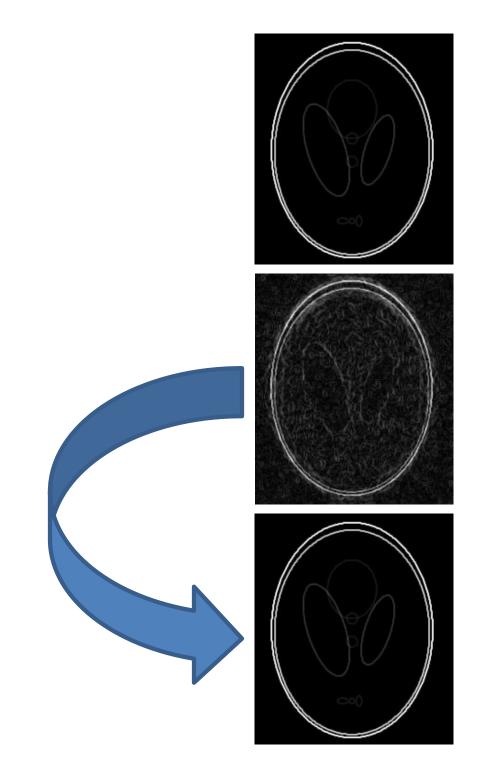


Compressed sensing

Compressibility/sparsity

Incoherence

Non-linear reconstruction



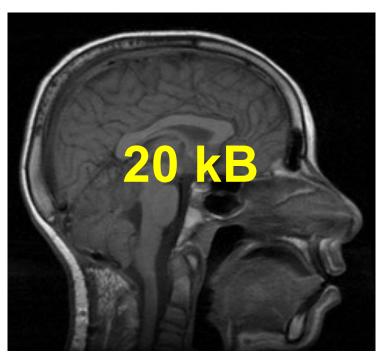
Medical images are compressible

Information < number of pixels



JPEG





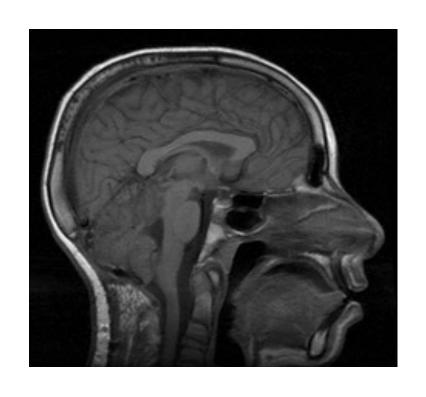
10% of the data → true information

90% of the data \rightarrow

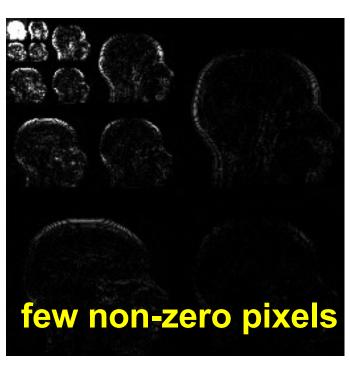


Medical images are compressible

Sparsity is the key



sparsifying transform wavelets (JPEG2000 standard)



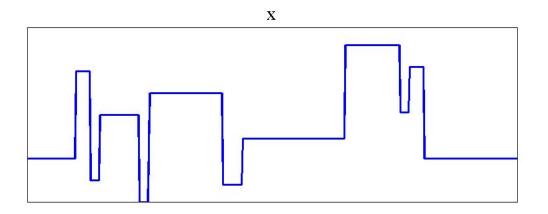
Sparsifying transforms

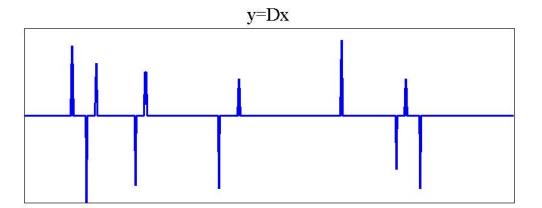
Finite differences

$$y(n) = x(n) - x(n-1)$$

In matrix form : y = Dx

$$D = \begin{bmatrix} 1 & -1 & & \\ & 1 & -1 & \\ & & 1 & -1 \end{bmatrix}$$





Total variation

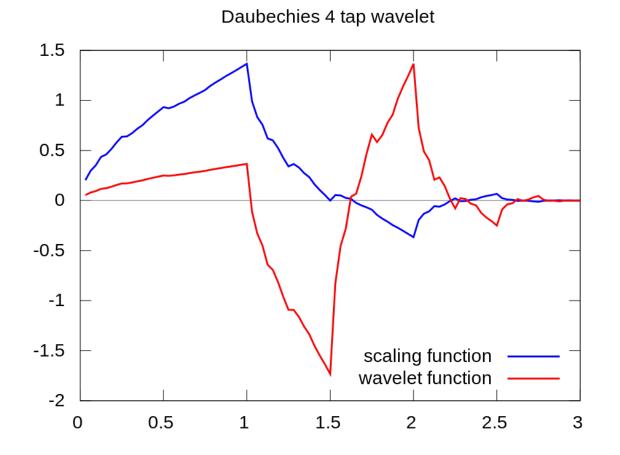
$$TV(x) = \sum_{n=2}^{N} |x(n) - x(n-1)| \qquad \qquad \min TV(x) = \min ||Dx||_{1}$$

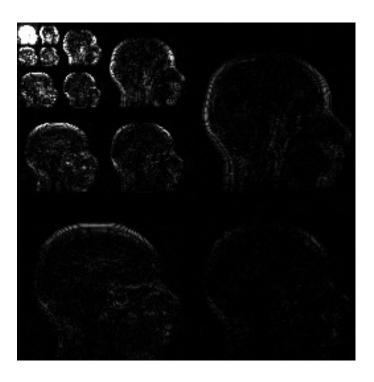


$$\min TV(x) = \min \|Dx\|_{1}$$

Sparsifying transforms

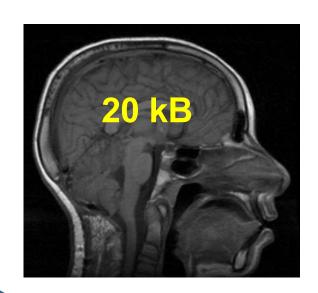
- Wavelets
 - Space-frequency localization
 - More efficient than Fourier transform to represent discontinuities





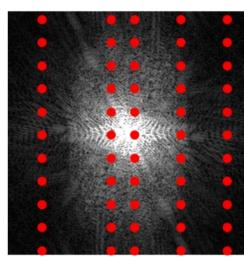
Compressed sensing

Exploit compressibility/sparsity to reduce k-space data



JPEG

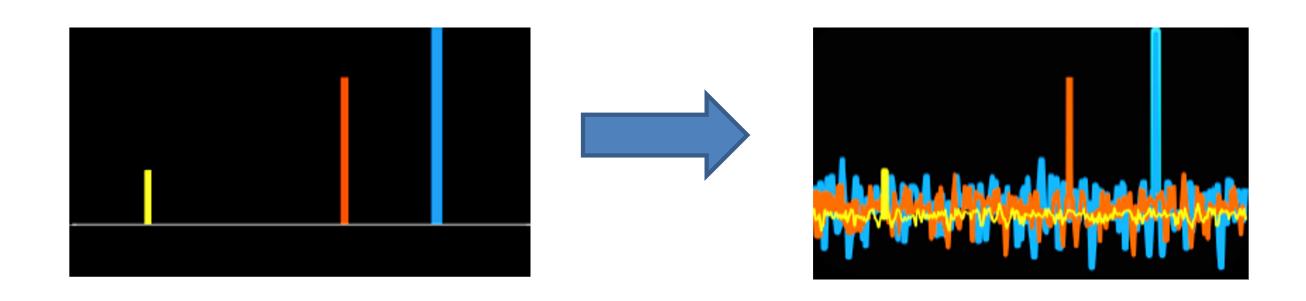
Compressed sensing 20 sec





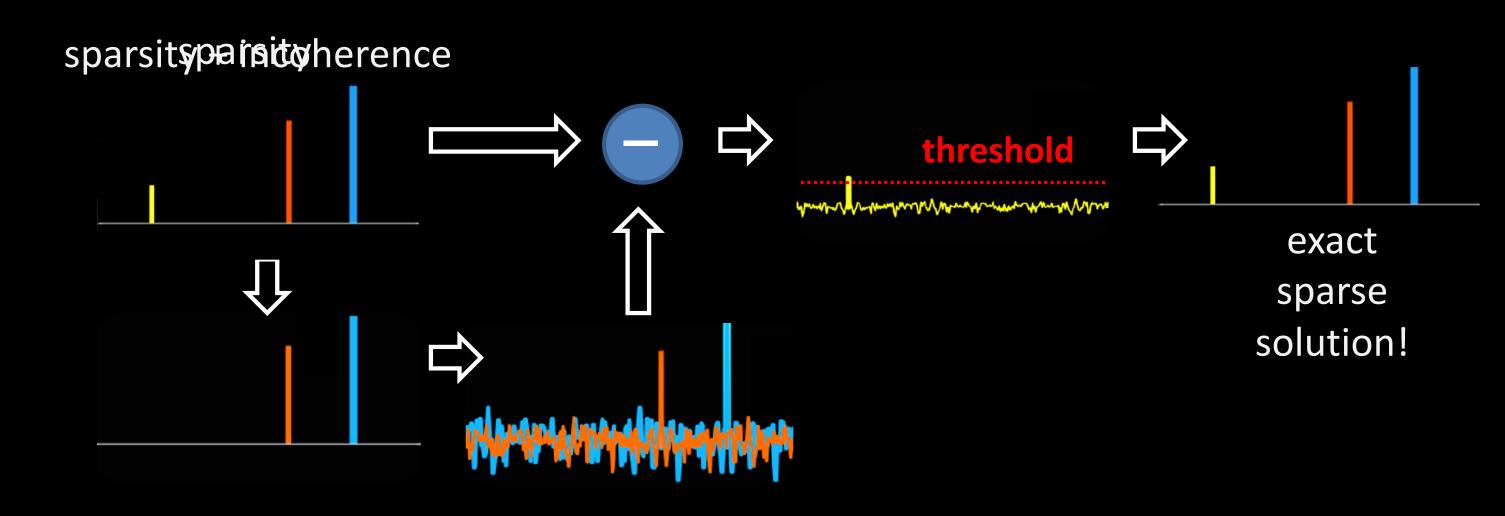
How to sample k-space to exploit image sparsity?

- Incoherence
 - Aliasing artifacts should not replicate image features
 - Aliasing artifacts should look like noise



How to reconstruct to exploit image sparsity?

Sparse reconstruction



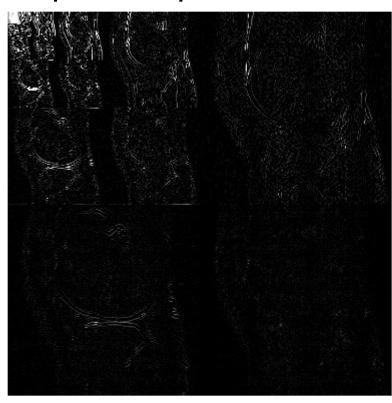
Incoherent k-space sampling

Knee image example

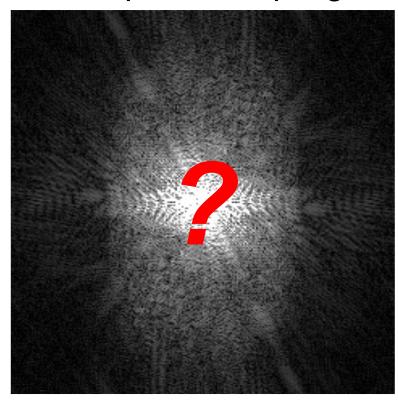
fully-sampled image



sparse representation



k-space sampling

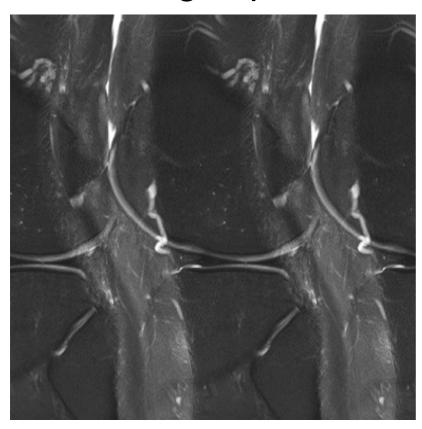


Incoherent k-space sampling

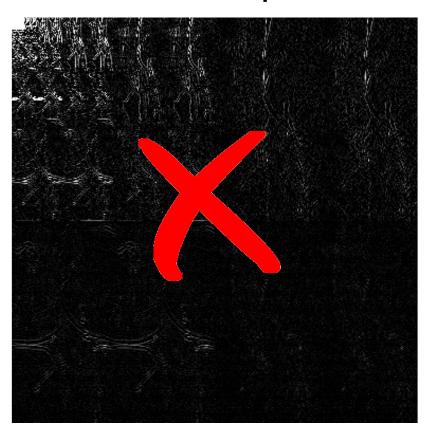
Uniform k-space undersampling

k-space

image space



transform space



sparsity is lost

Incoherent k-space sampling

Non-uniform k-space undersampling

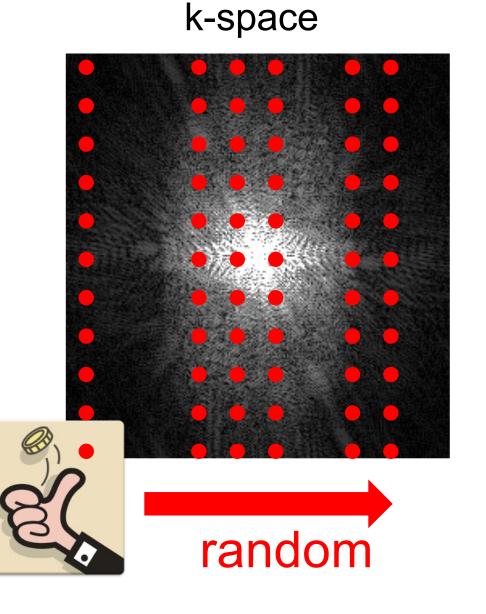
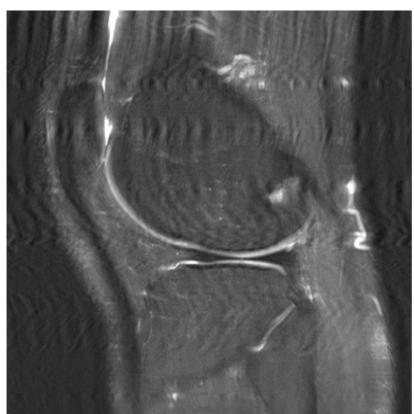


image space



transform space



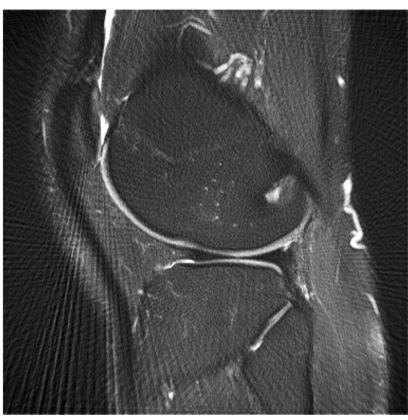
sparsity is preserved

Inherent incoherence in non-Cartesian sampling

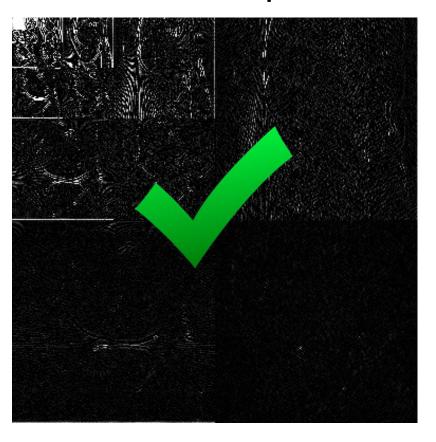
Radial sampling

k-space

image space



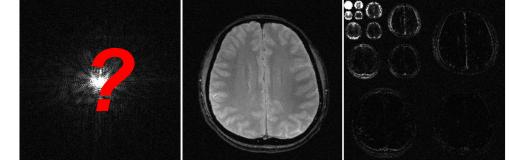
transform space



sparsity is preserved

Incoherent sampling patterns

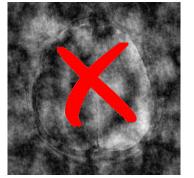
What are the best samples?

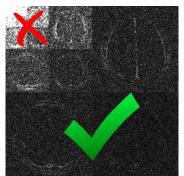


sampling pattern image domain sparse domain

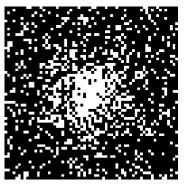
random undersampling (Cartesian)

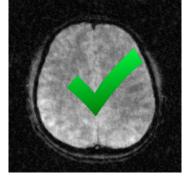


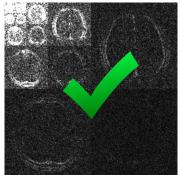




variable-density random undersampling (Cartesian)

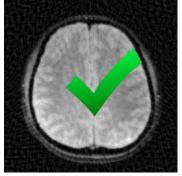


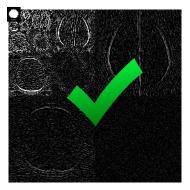




regular undersampling (radial)







Sparse reconstruction

Optimization problem, iterative algorithm

Acquisition model: d = Emd: k-space data
E: undersampled Fourier transform
m: image to reconstruct

$$\min_{\mathbf{m}} \left\| \mathbf{Em} - \mathbf{d} \right\|_{2}^{2} + \lambda \left\| \mathbf{Tm} \right\|_{1}$$

- T: sparsifying transform
- **λ: regularization parameter**
 - Trade-off between sparsity (I1-norm term) and data consistency (I2norm term)

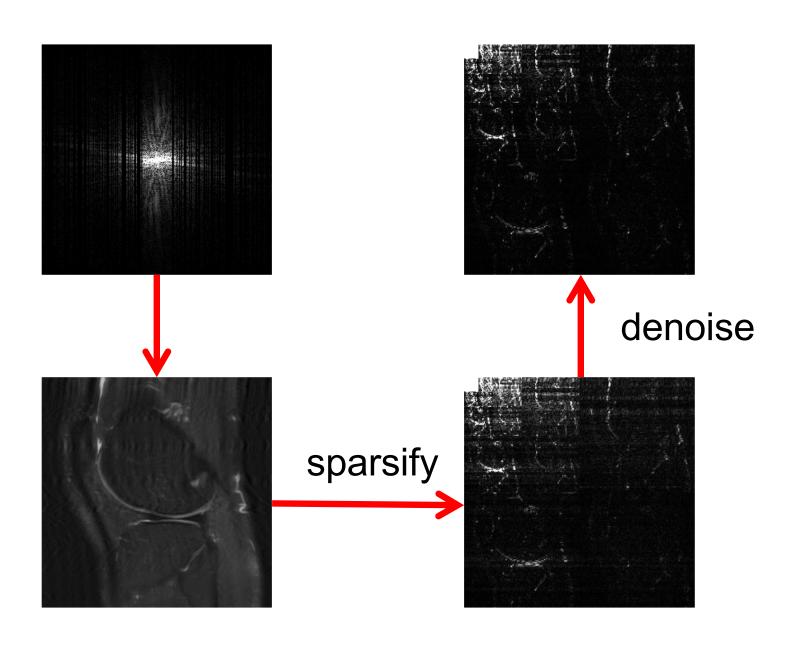
```
inputs: E, d, T, \lambda
```

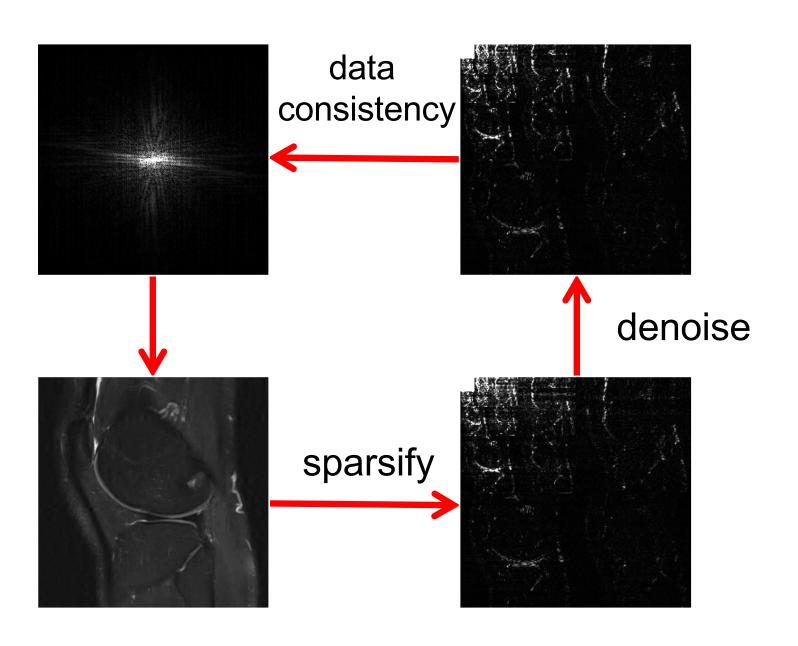
initial solution: $m_0 = E^*d$

for each iteration k

```
enforce sparsity: m_k = T^* \big( Soft(Tm_{k-1}, \lambda) \big) enforce data consistency: m_k = m_k - E^* (Em_k - d)
```

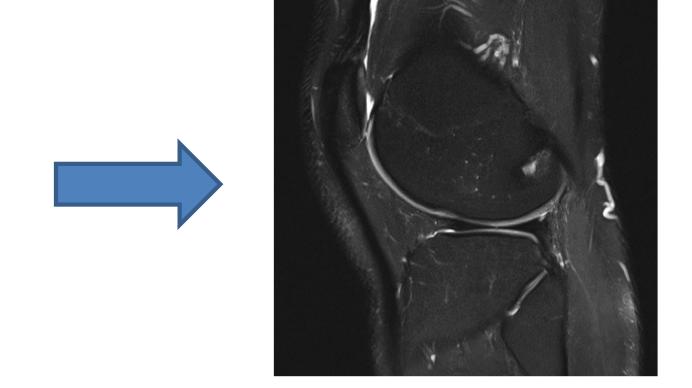
end for





initial solution

after 40 iterations

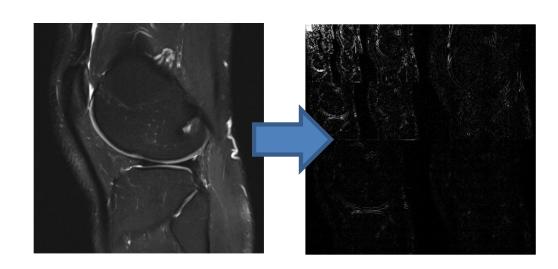


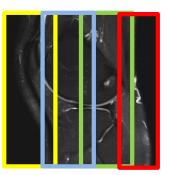
Combination of compressed sensing and parallel imaging

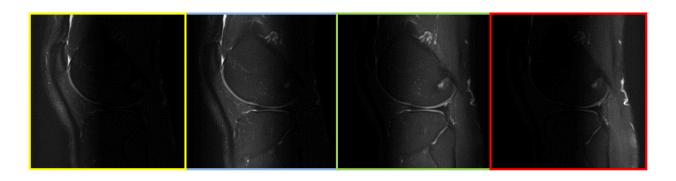
Complementary sources of information

sparsity

coil sensitivity encoding

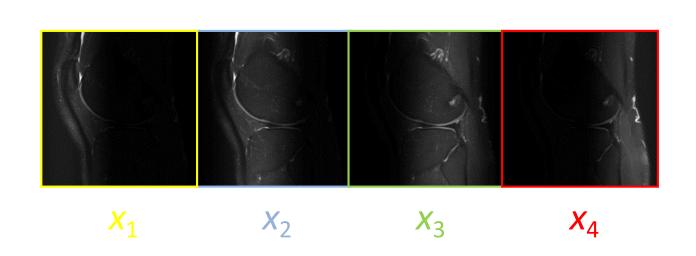






Combination of compressed sensing and parallel imaging

- Joint sparsity rather than coil-by-coil sparsity
 - New definition of sparsity
 - Exploit inter-coil correlations



$$x = [x_1, x_2, x_3, x_4]$$

(1) joint
$$I_1$$
-norm: $\|\mathbf{x}_{MF}\|_1 = \sum_{p=1}^{Np} \left| \sum_{l=1}^{Nc} \frac{c_l^*(p) x_l(p)}{|c_l(p)|^2} \right|$

(2)
$$I_1 - I_2$$
-norm: $\|\mathbf{x}\|_{1,2} = \|x_{SOS}\|_1 = \sum_{p=1}^{Np} \left(\sum_{l=1}^{Nc} |x_l(p)|^2\right)^{1/2}$

- (1) Otazo R et al. ISMRM 2009, 378; MRM 2010
- (2) Lustig M et al. ISMRM 2009, 379; MRM 2010

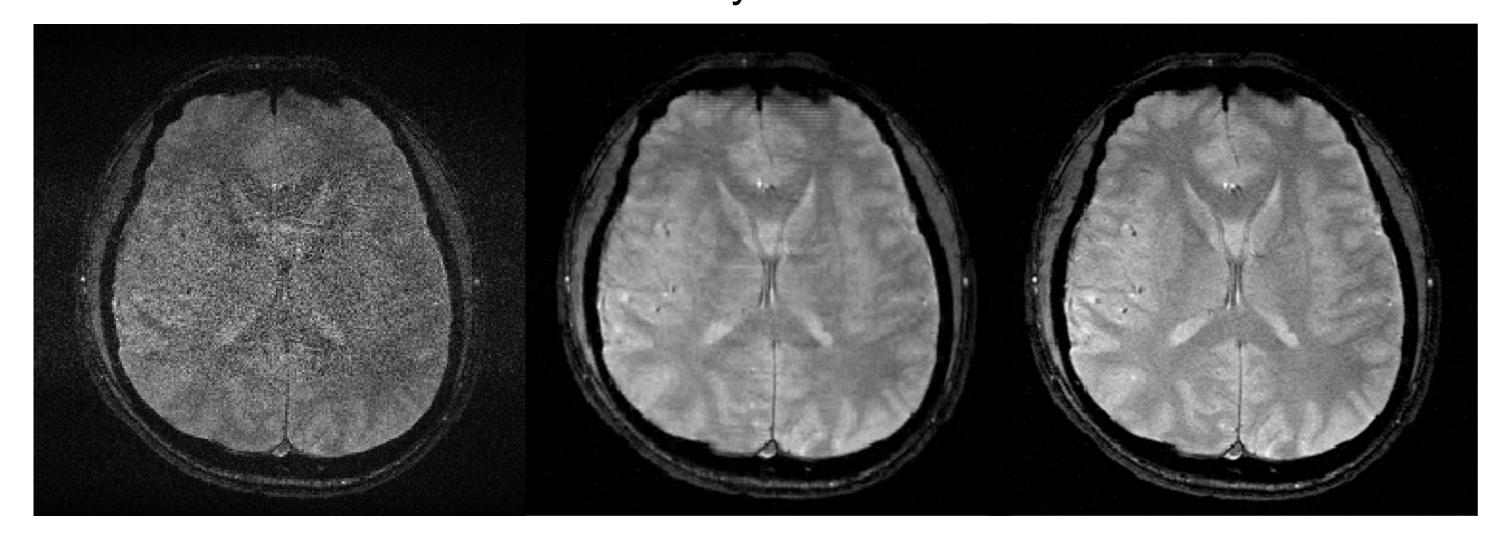
SPARSE-SENSE

4-fold acceleration

GRAPPA

Coil-by-coil CS

SPARSE-SENSE

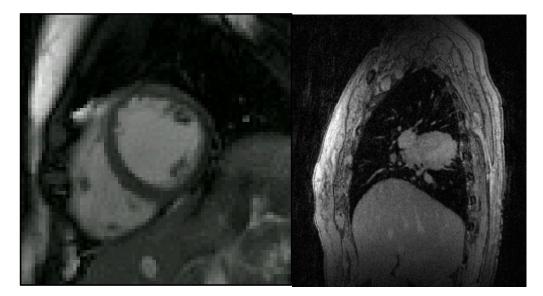


Otazo R et al. ISMRM 2009, 378

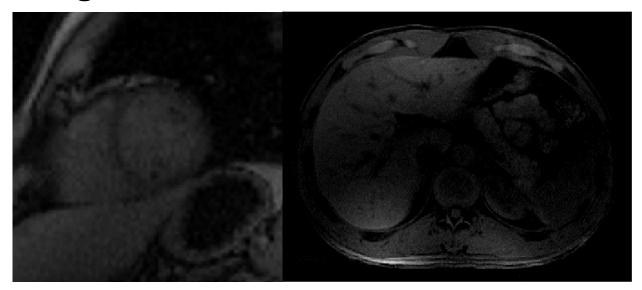
Dynamic MRI

- Time-series of images
- Video
- Physiological information

motion

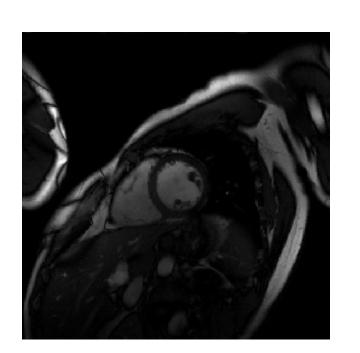


contrast agent
- gadolinium-based: T1 reduction

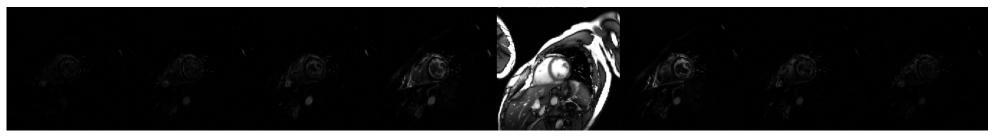


Temporal compressed sensing

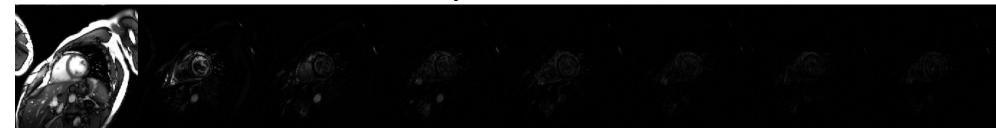
- Temporal sparsity
 - Videos are more compressible than images





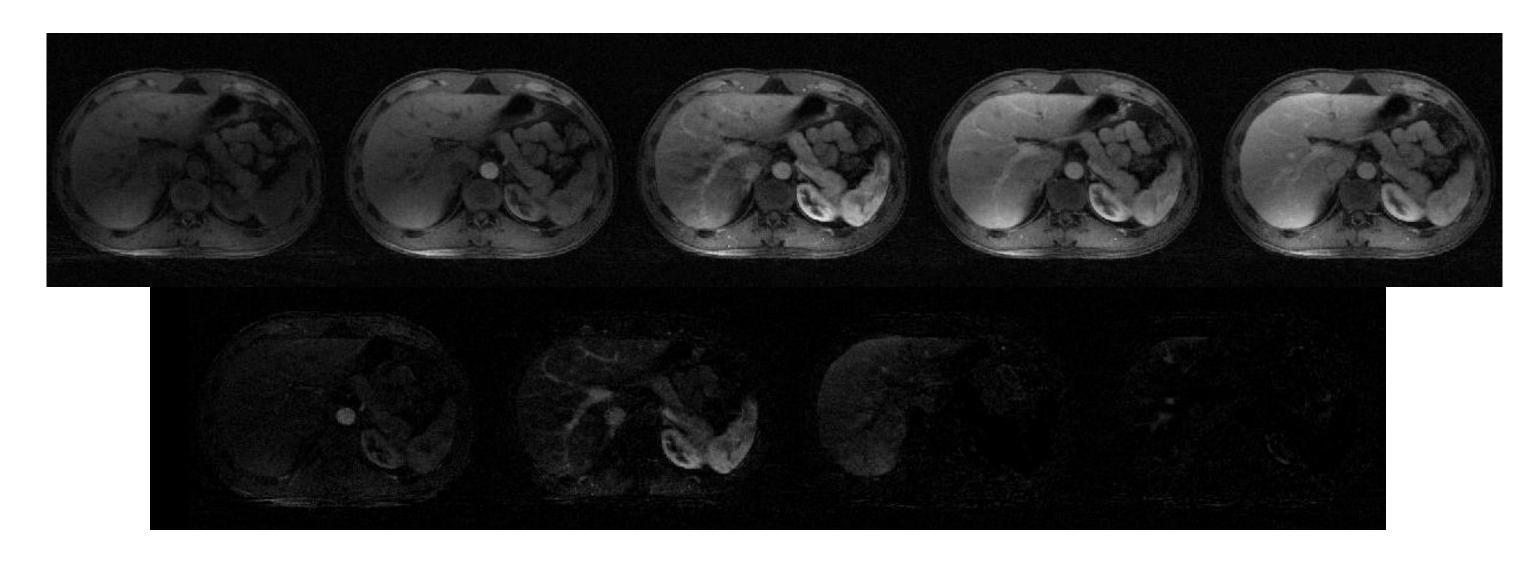


Temporal PCA



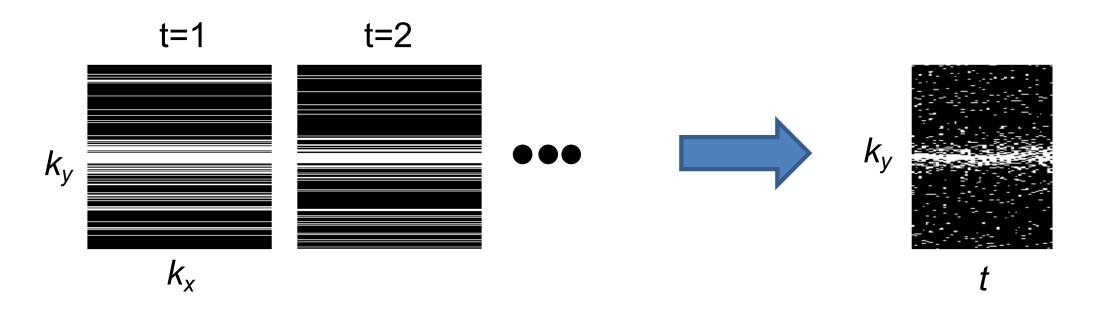
Temporal compressed sensing

- Temporal sparsity
 - Difference between consecutive frames is sparse



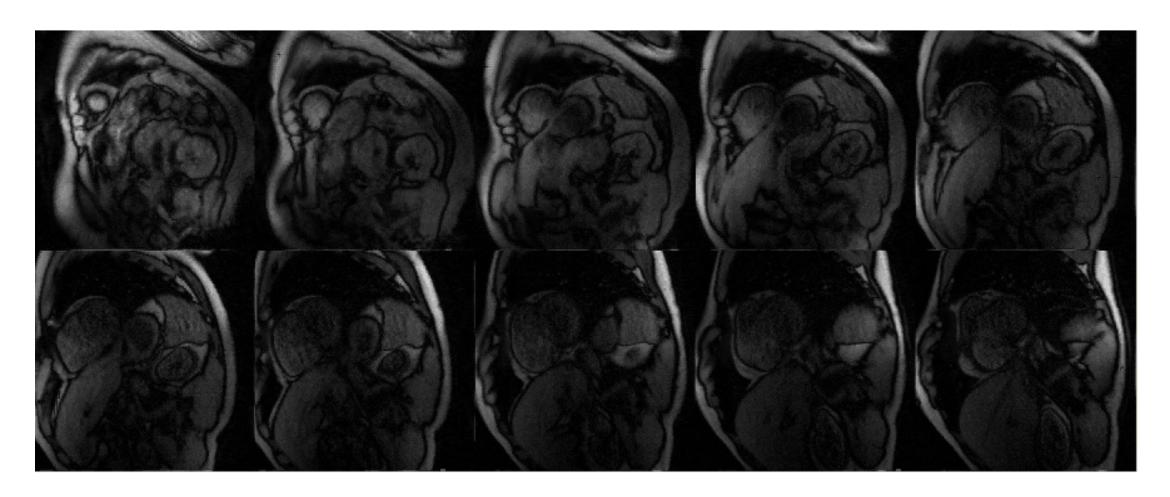
Temporal compressed sensing

- Temporal incoherence
 - Different irregular k-space sampling pattern for each time point
 - Random Cartesian



k-t SPARSE-SENSE

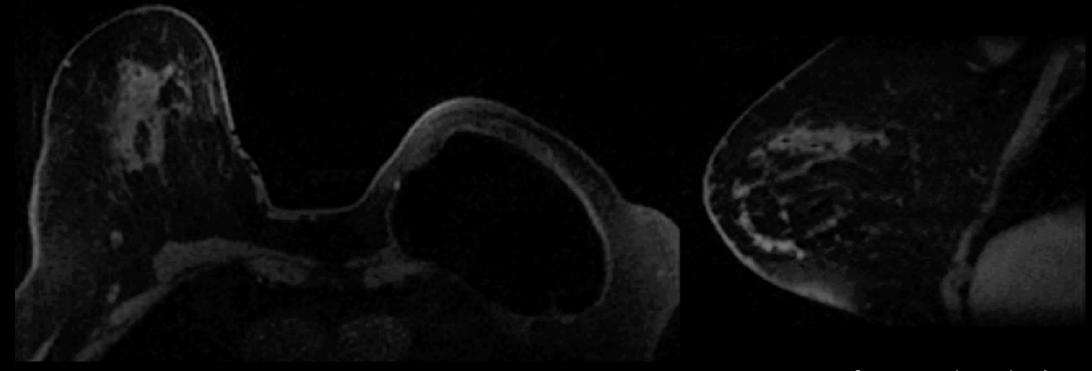
- Cardiac perfusion (video of contrast enhancement)
 - 8-fold acceleration, 10 slices per heartbeat
 - Temporal res. = 60 ms/slice, spatial res. = $1.7x1.7x3 \text{ mm}^3$



GRASP: radial compressed sensing

- Dynamic contrast enhanced imaging
- Isotropic spatial resolution = 1.6 mm³
- Temporal resolution = 5 seconds

78-year-old woman with stage T1c invasive ductal carcinoma

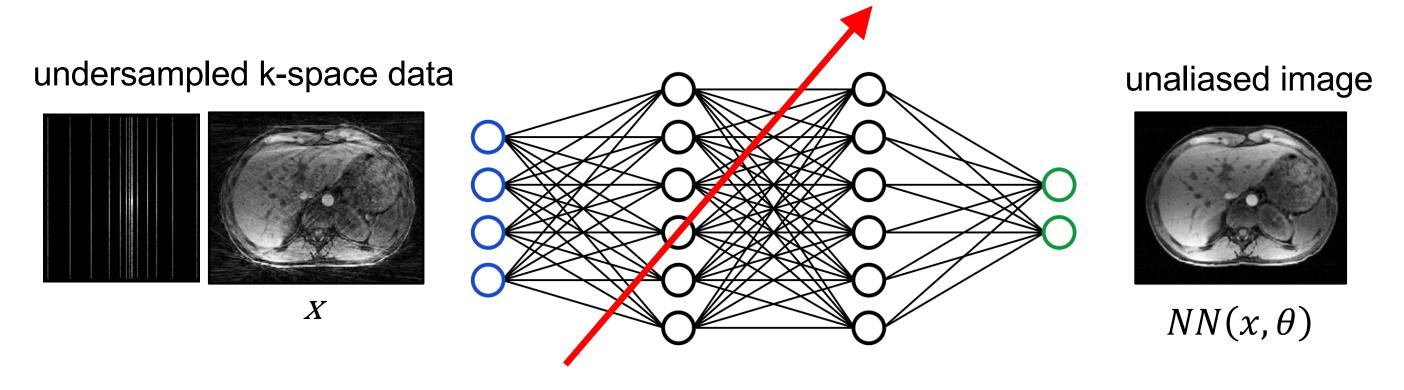


acquired axial orientation

reformatted sagittal

Deep learning MRI reconstruction

 Neural network for reconstruction for undersampled kspace data



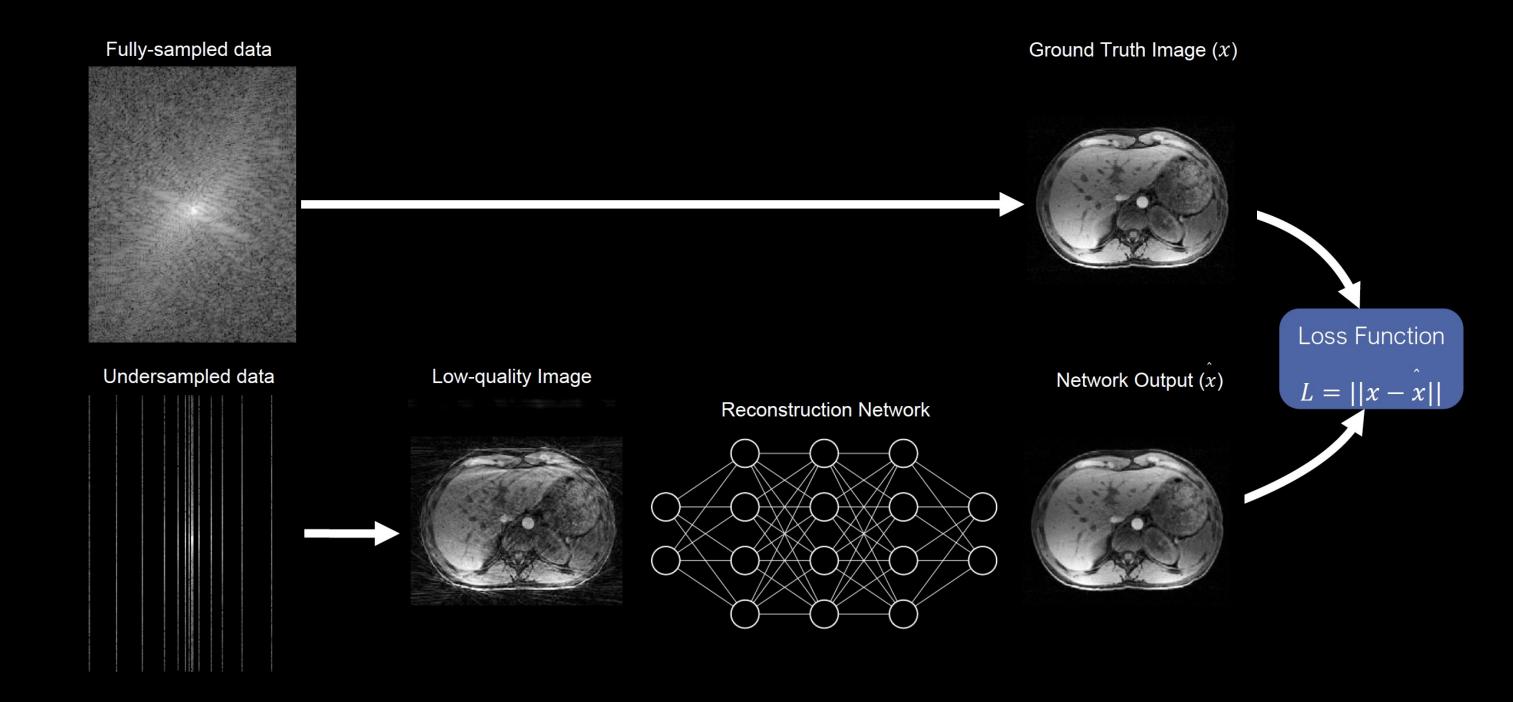
learning: compute network parameters to minimize loss function

$$L(\theta) = \sum_{i} ||NN(x_i, \theta_i) - y_i||$$

y: training target

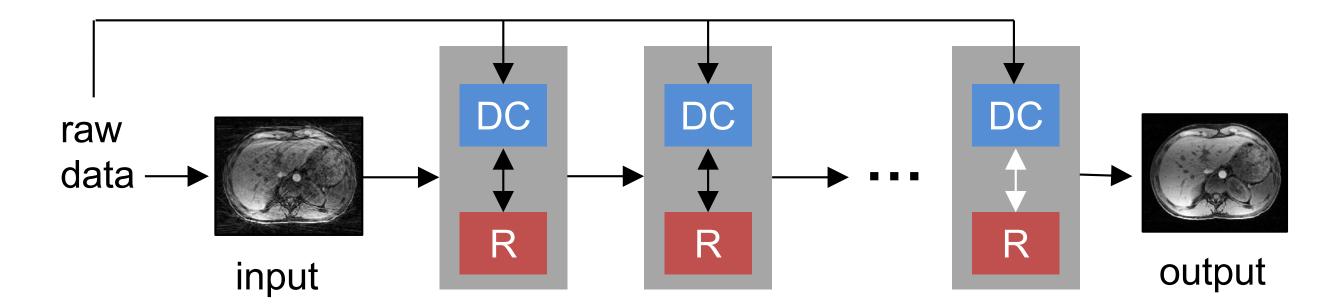
i: index for training cases

Supervised learning with fully-sampled k-space data



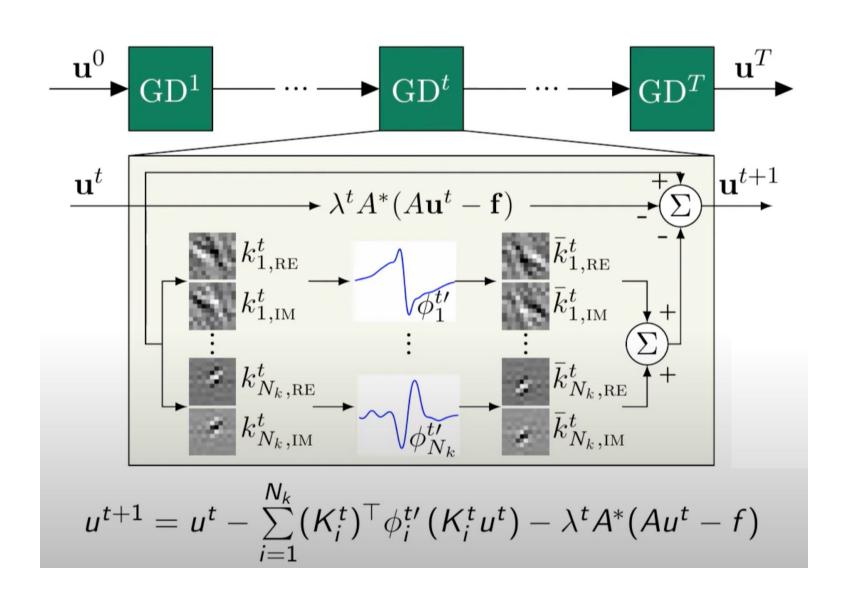
Unrolled network

- Unroll compressed sensing iterations
 - Fixed data consistency (DC)
 - Trainable regularization (R)



Variational network

Gradient descent algorithm as a network

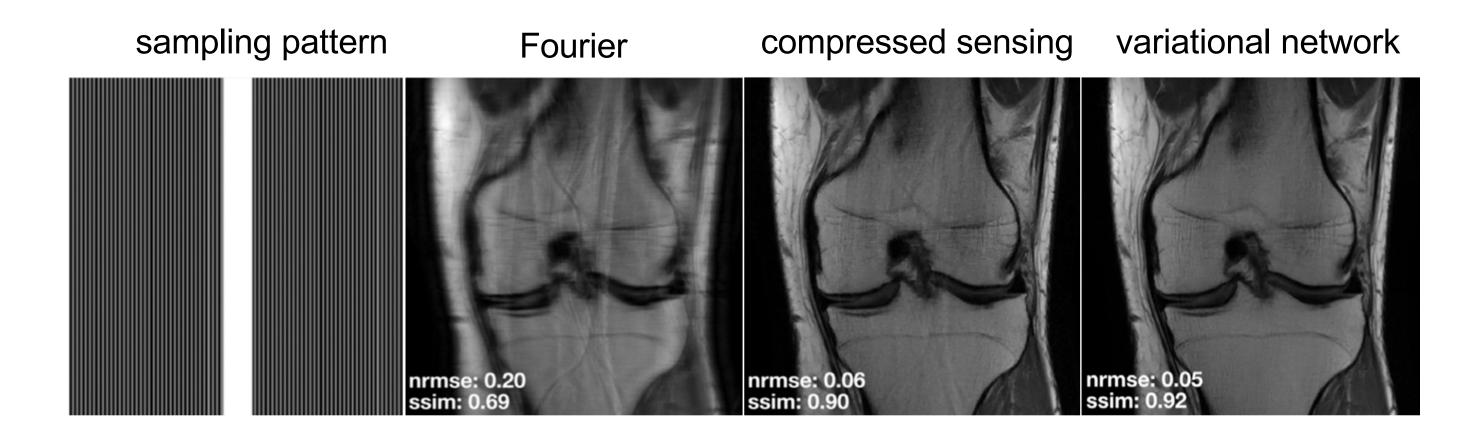


A: acquisition model

Hammernik K et al. MRM 2017

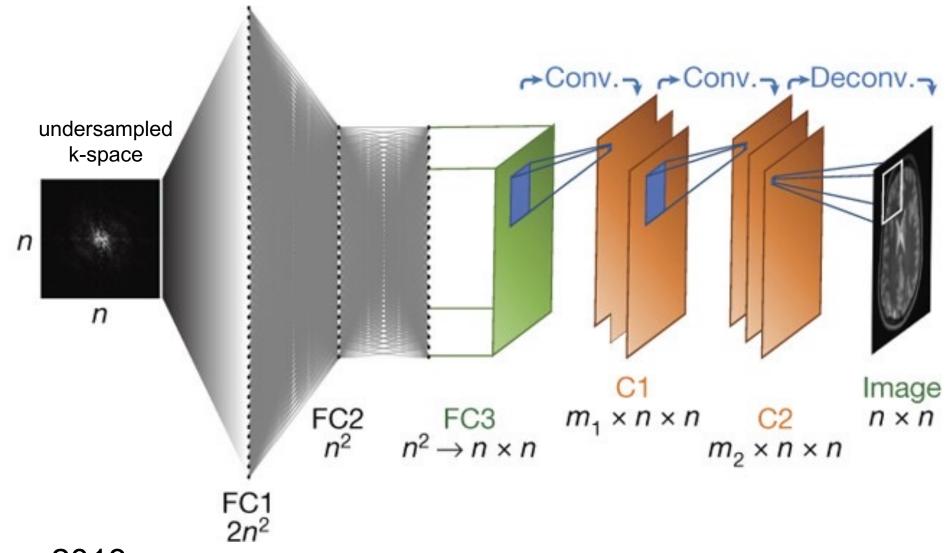
Variational network

4-fold acceleration



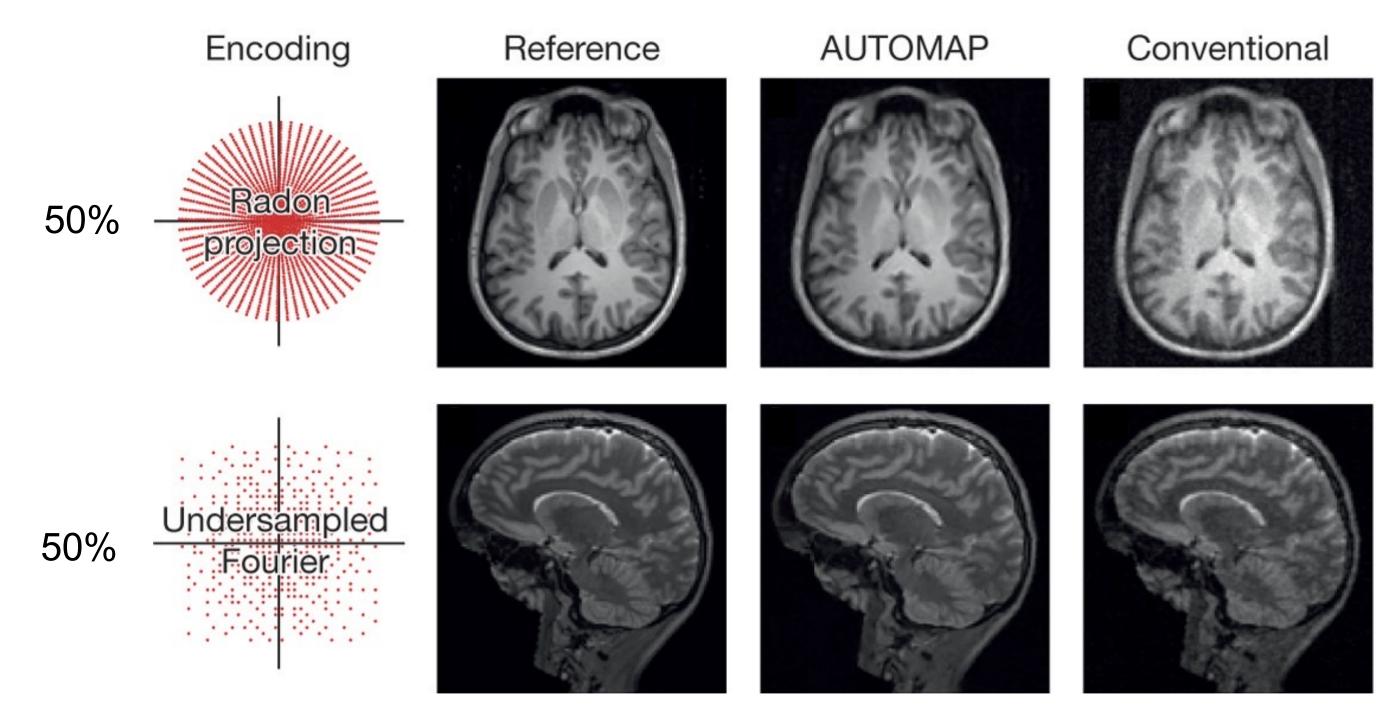
AUTOMAP: self-consistent network

- Fully-connected + convolutional layers
 - Low-dimensional manifold learning



Zhu B et al. Nature 2018

AUTOMAP

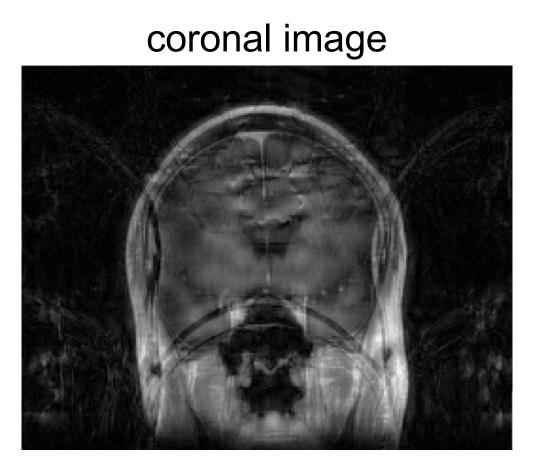


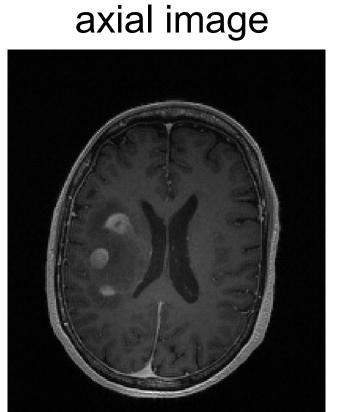
Zhu B et al. Nature 2018

Modular network for 3D MRI with 2D acceleration

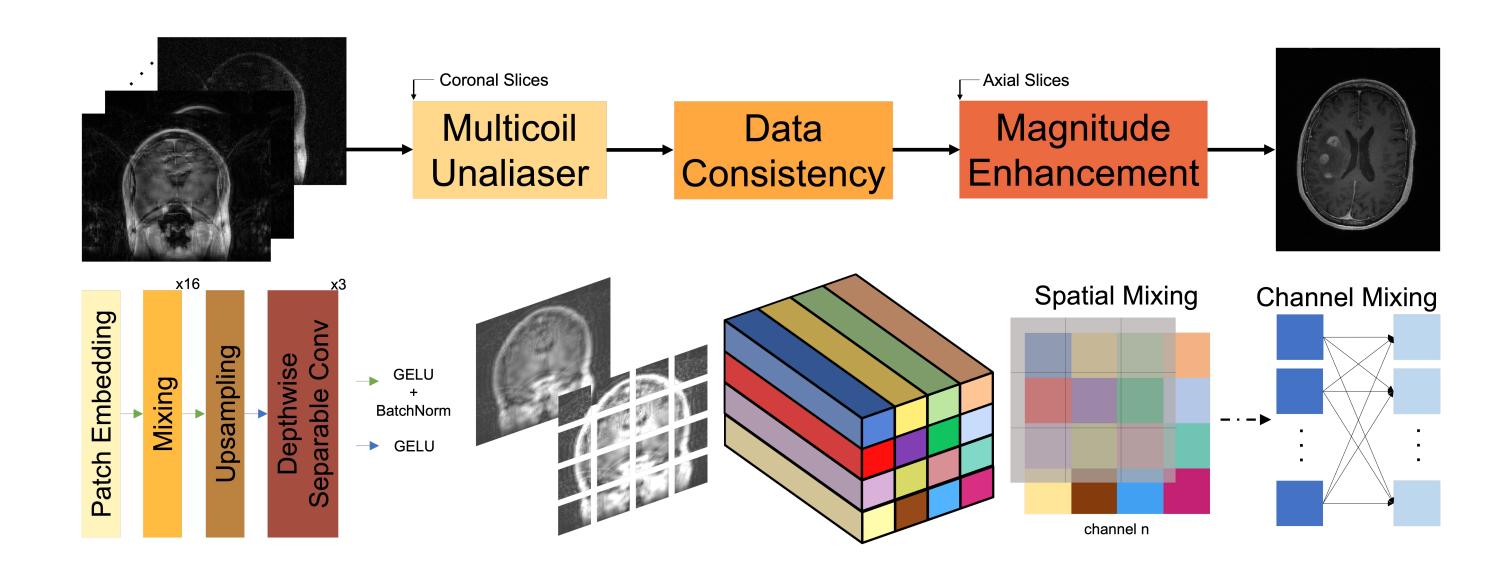
Brain MRI: k_y-k_z acceleration (coronal), axial image evaluation

8-fold acceleration



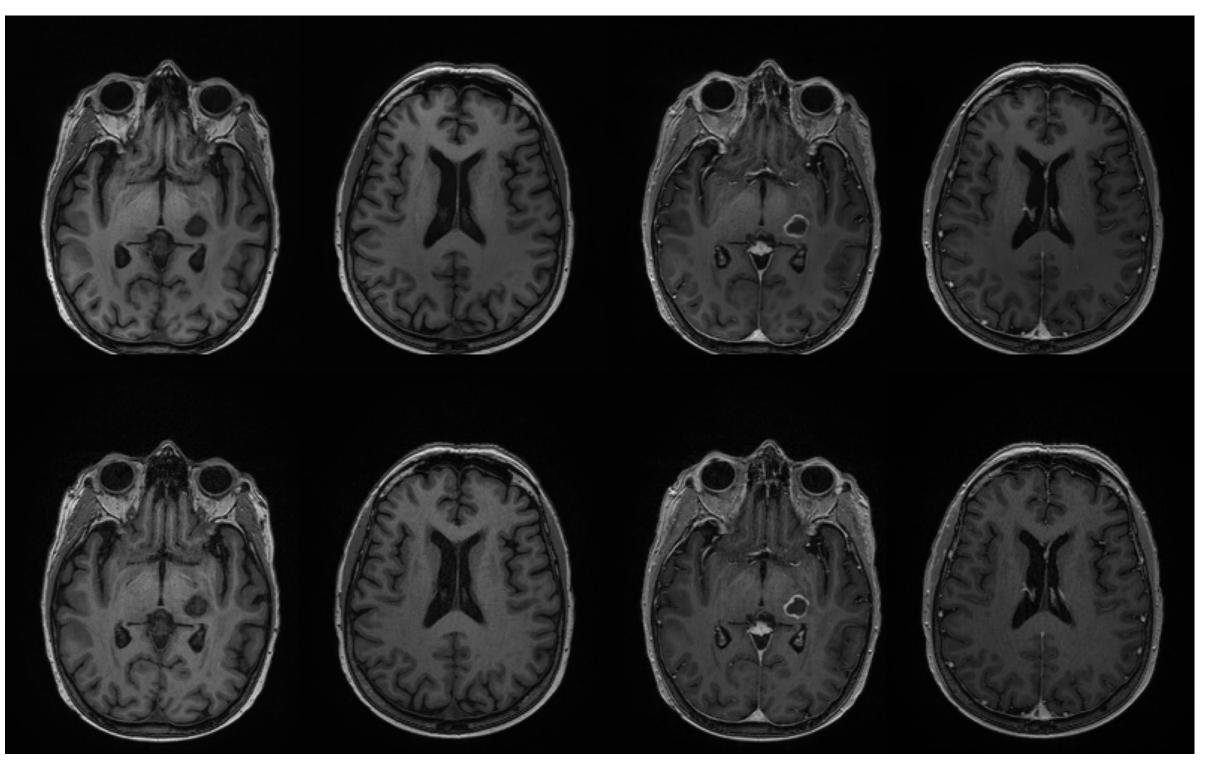


Modular network for 3D MRI with 2D acceleration



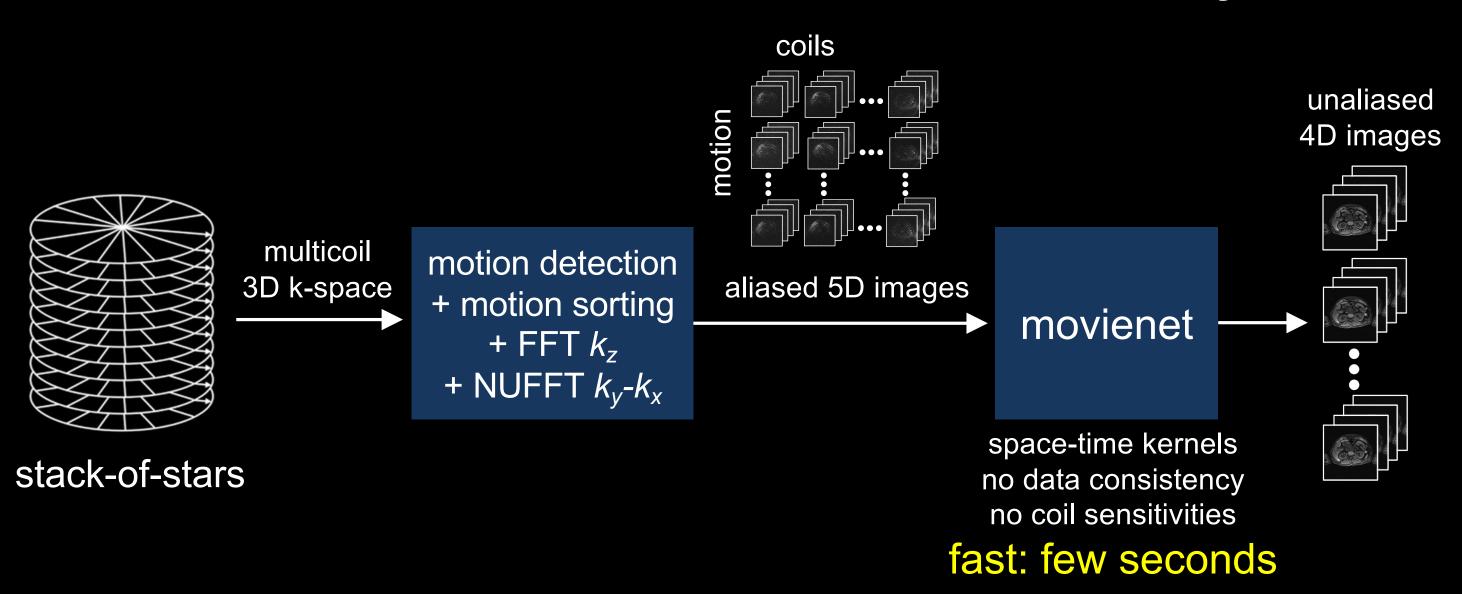
Deep learning
Scan time:
2 x 90 sec

Compressed sensing
Scan time:
2 x 166 sec



movienet

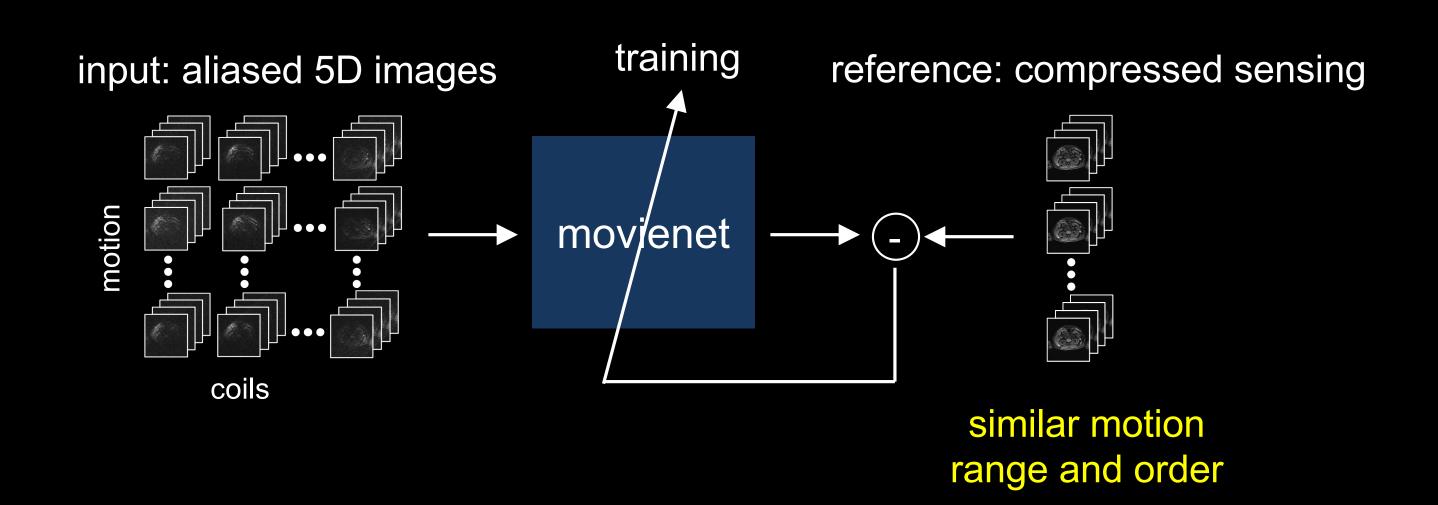
Fast 4D reconstruction without data consistency



Murray V et al. MRM 2023

Motion consistency instead of data consistency

Preserve motion range and order of motion states in the input

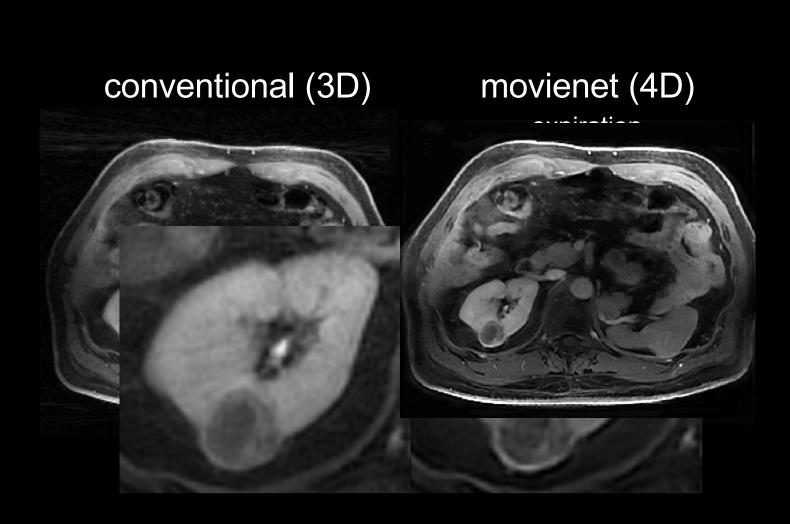


Motion-tolerant MRI with movienet

- Scan time = 1 minute
- movienet with 4 motion states
- Live mode picture

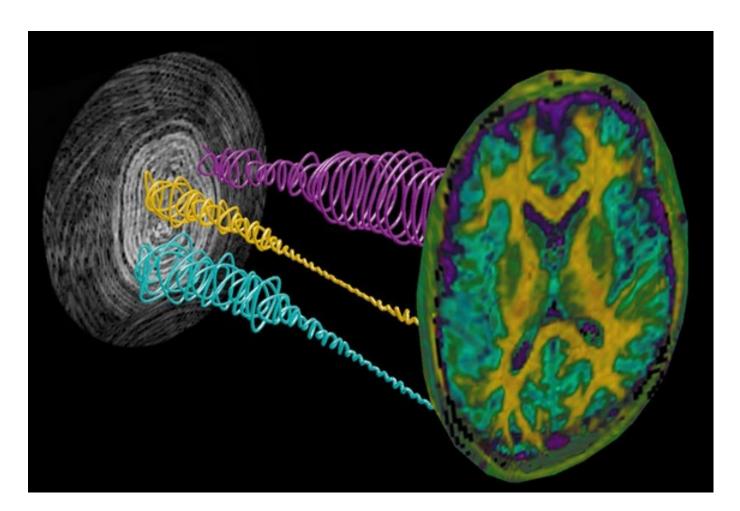


patient with kidney cyst

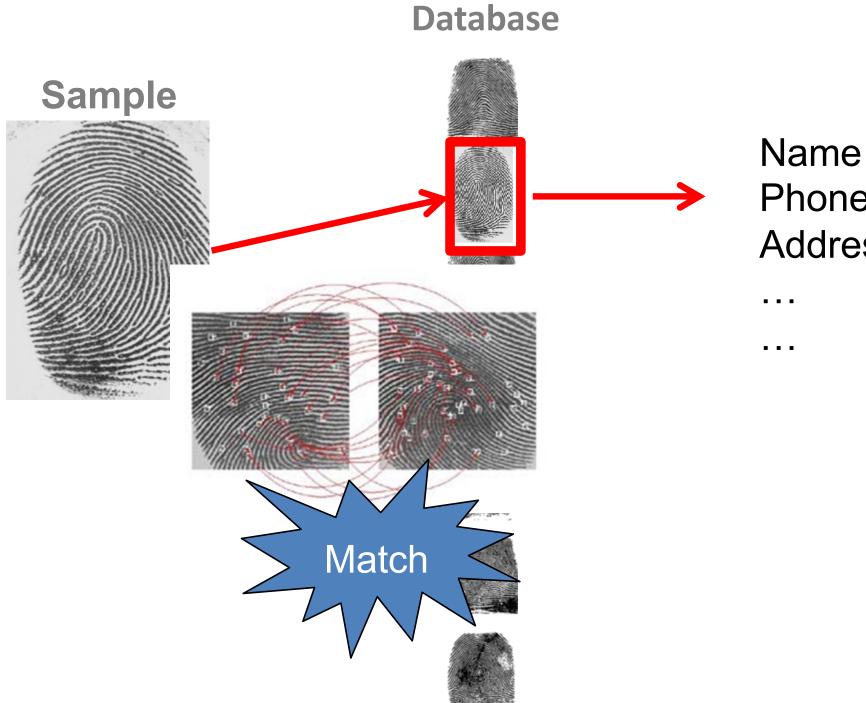


MR fingerprinting

- Fast quantitative mapping of tissue parameters
 - T1, T2, PD
 - Diffusion
 - pH level
- Different types of tissue have unique MR signal evolutions

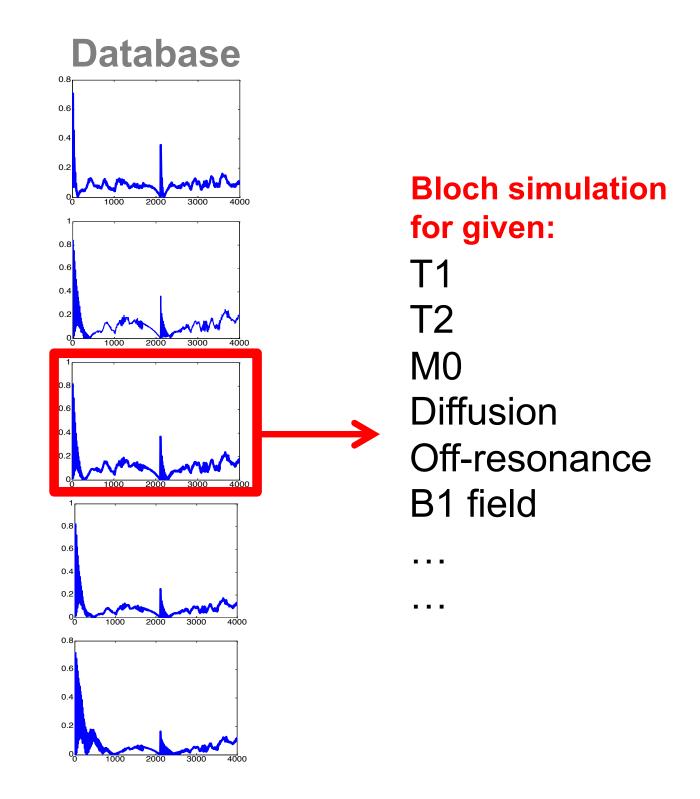


Fingerprint identification system

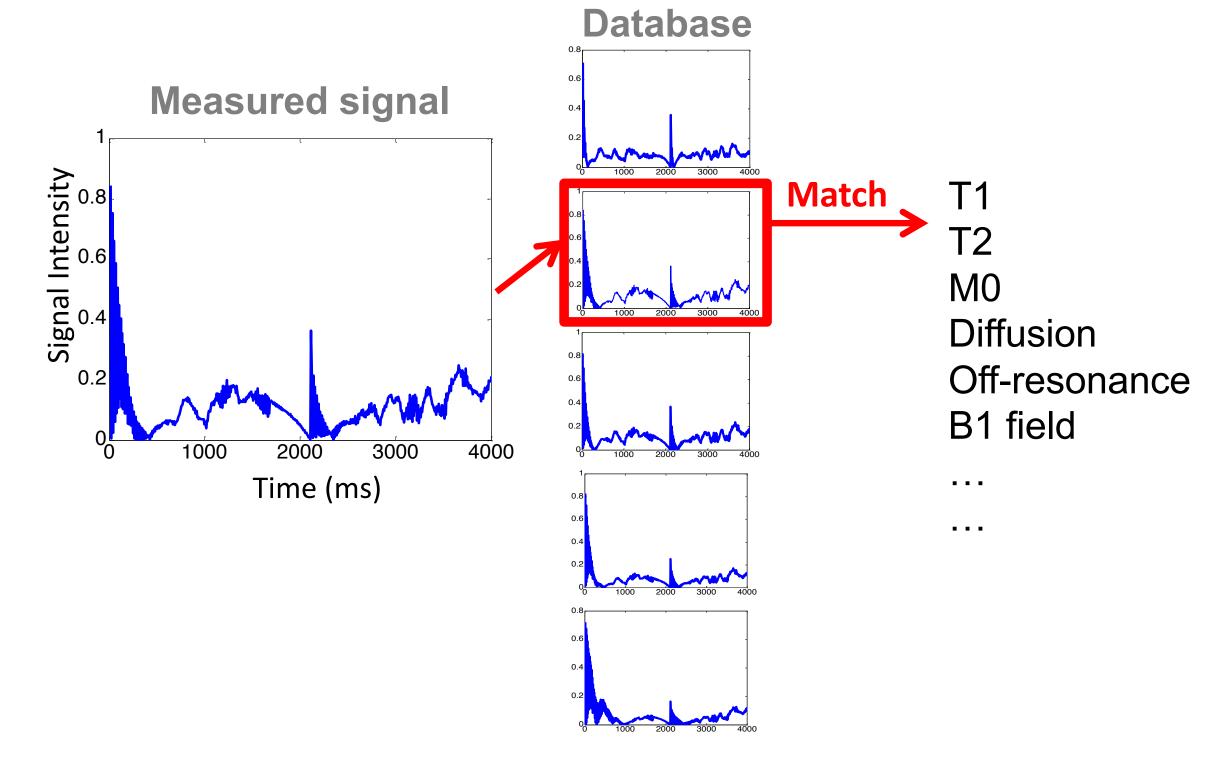


Name
Phone number
Address

MR fingerprinting

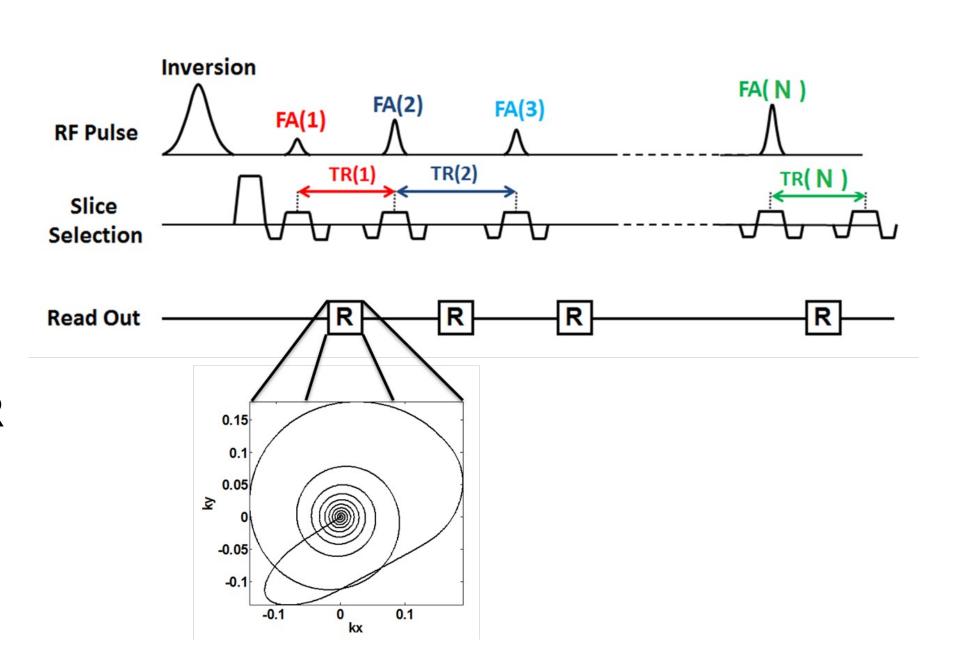


MR fingerprinting



MR fingerprinting acquisition

- Series of highlyundersampled data
- Each frame has mixed contrast
 - Different FA and TR



MR fingerprinting reconstruction

Database entry that yields highest correlation with the input temporal signal

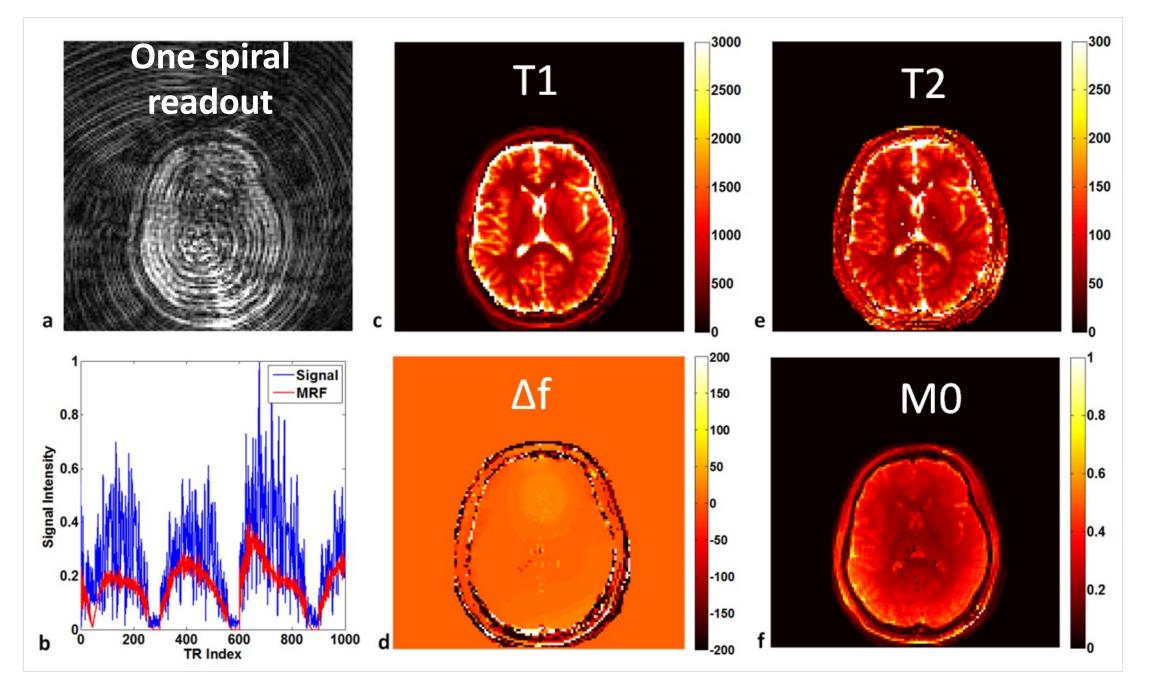
$$i(r) = \arg\max_{i} \langle d_i, s(r) \rangle$$

i(r): database index for pixel r

 d_i : *i*-th database element

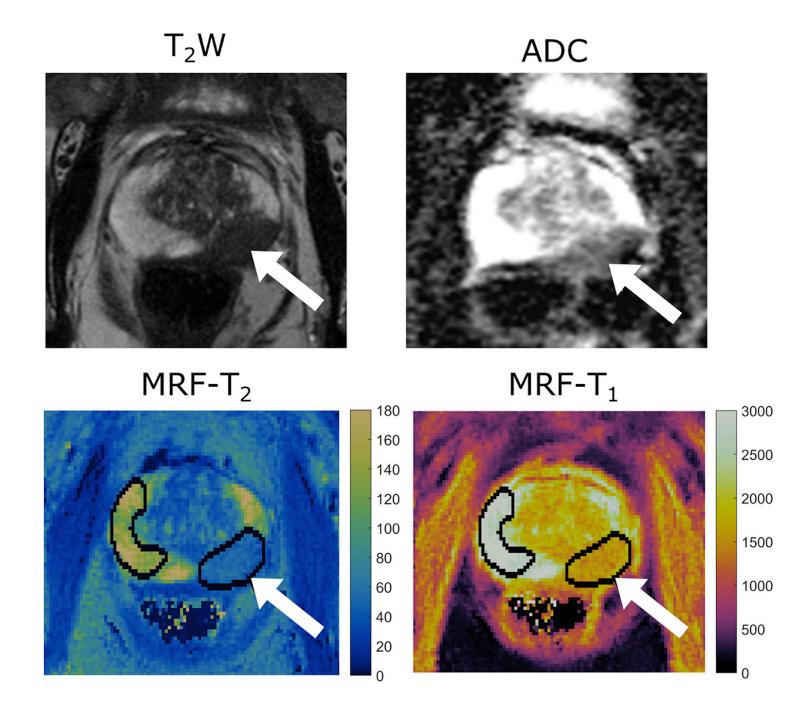
s(r): temporal signal for pixel r

MR fingerprinting in the brain



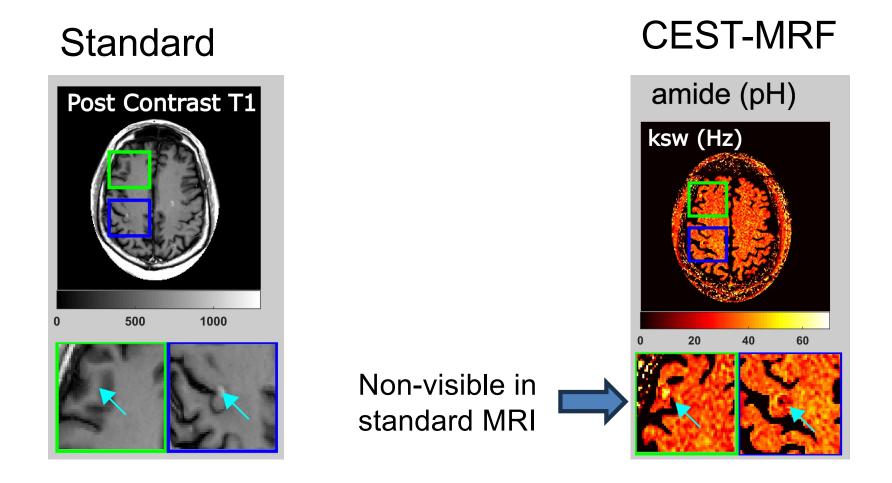
1,000 time frames, scan time = 12.3 seconds

MR fingerprinting of prostate cancer



Metabolic MR fingerprinting (CEST-MRF)

- New contrast beyond T1 and T2 mapping: pH level of tumors
- Improved diagnosis and treatment monitoring of brain tumors



Review paper on compressed sensing



A look at how CS can improve on current imaging techniques

Michael Lustig, David L. Donoho, Juan M. Santos, and John M. Pauly ompressed sensing (CS) aims to reconstruct signals and images from significantly fewer measurements than were traditionally thought necessary. Magnetic resonance imaging (MRI) is an essential medical imaging tool with an inherently slow data acquisition process. Applying CS to MRI offers potentially significant scan time reductions, with benefits for patients and health care economics.

 What are the factors that limit the speed of conventional MRI using magnetic field gradients?

- Gradient performance: amplitude and slew-rate
 - Physical: power increases with the cube of the acceleration factor
 - Physiological: peripheral nerve stimulation due to fast gradient switching

What are the main ingredients for compressed sensing?

- Sparsity: image representation
- Incoherence: k-space undersampling pattern
- Non-linear reconstruction: promote sparsity

 Why is MRI a good candidate for application of compressed sensing?

- Images are compressible (in general)
- Acquisition is in a transform domain (facilitates incoherence)
 - Software change to undersample k-space

Is random k-space sampling the only way to achieve incoherence?

 No, regular undersampling of non-Cartesian k-space trajectories will also result in incoherence

 Can you apply compressed sensing to accelerate the acquisition of dynamic MRI (videos)? Discuss the main differences with respect to acceleration of anatomical images

Yes, in fact videos are more compressible than images and therefore higher accelerations should be expected for dynamic imaging. The main differences would be to have a sparsifying transform that exploits temporal correlations and to change the k-space undersampling pattern for each time point to have spatial and temporal incoherence Hands-on exercise: compressed sensing in Matlab

Homework: deep learning reconstruction