1. Integrating by parts, we have $\int_0^\infty S(t)dt = tS(t)|_{t=0}^\infty + \int_0^\infty tdF(t)$. Thus it suffices to show that $\lim_{t\to\infty} tS(t) = 0$. This can be done by using the dominated convergence theorem or simply by observing that, as $M \to \infty$, $\epsilon > \int_{t=M}^\infty tdF(t) \geq M\int_{t=M}^\infty dF(t) = M(1-F(M))$ for any given $\epsilon > 0$. The first inequality is due to the assumption that $E(T) < \infty$.

   An alternative proof is to use the Fubini Theorem as below.

   $\int_0^\infty S(t)dt = \int_0^\infty \int_t^\infty dF(u)dt = \int_0^\infty \int_0^u dt\, dF(u) = \int_0^\infty udF(u)$.

2. First note that $\frac{\Delta X}{G(X)} = \frac{\Delta T}{G(T)}$. Using a conditioning argument, we have $E\Big[\frac{\Delta T}{G(T)}\Big] = E\Big\{E\Big[\frac{I(C\geq T)T}{G(T)}\Big|T\Big]\Big\} = E\Big\{\frac{T}{G(T)}E\Big[I(C\geq T)|T\Big]\Big\} = E\{\frac{T}{G(T)}G(T)\} = E(T)$.

   An alternative proof is to use the joint distribution of T and C (which are independent) as follows. You will see that it essentially follows the same idea as above.

   $E\Big[\frac{\Delta X}{G(X)}\Big] = \int_0^\infty \int_t^\infty d(-G(c))\frac{t}{G(t)}\,dF(t) = \int_0^\infty G(t)\frac{t}{G(t)}\,dF(t) = \int_0^\infty t\,dF(t) = E(T)$.

3. We may start with $S(t|Z=z)$:

$$\begin{aligned} S(t|Z=z) &= P(T>t|Z=z) = P\Big[h(T) > h(t)|Z=z\Big] \\ &= P\Big[\epsilon > h(t) - \beta' z\Big] = D\Big[h(t) - \beta' z\Big] . \end{aligned}$$

   Applying D^{-1} to both sides, we get (i). (ii) follows trivially by rearranging terms: $S(t|Z=z) = \Big[e^{-e^{h(t)}}\Big]^{e^{-\beta' z}} := S_0(t)^{e^{-\beta' z}}$, and then taking logarithm of both sides (recall that $\log S(t) = -\Lambda(t)$) and differentiate with respect to t.

4. To avoid the messy notation in Cox (1972), and to generalize the result, we will prove the follow claim:

   Suppose there are n independent continuous survival random variables $T_1, ..., T_n$ with hazard functions $\lambda_1(t), ..., \lambda_n(t)$. Their survival functions and density functions are denoted by $S_1(t), ..., S_n(t)$ and $f_1(t), ..., f_n(t)$, respectively (recall that $\lambda(t) = f(t)/S(t)$). Given the fact that the first failure occurred at time t_0, the conditional probability that this failure was contributed by the subject k is $\lambda_k(t)/(\lambda_1(t) + ... + \lambda_n(t))$.

   Proof: Consider the interval $[t_0, t_0 + dt)$ (later we will let $dt \to 0$). We know that one of the n subjects failed within this interval, and the rest survived beyond $t_0 + dt$. So the conditional probability that this subject happens to be k is

$$\frac{P\Big(T_k \in [t_0, t_0+dt); \quad T_j \geq t_0 + dt \ \ \forall j \neq k\Big)}{P\Big(T_i \in [t_0, t_0+dt) \ \ for \ \ some \ \ i \in \{1, ..., n\}; \quad T_j \geq t_0 + dt \ \ \forall j \neq i\Big)}$$

The numerator above can be expressed as

$$S_1(t_0+dt)\Big[S_k(t_0)-S_k(t_0+dt)\Big]...S_n(t_0+dt)\,,$$

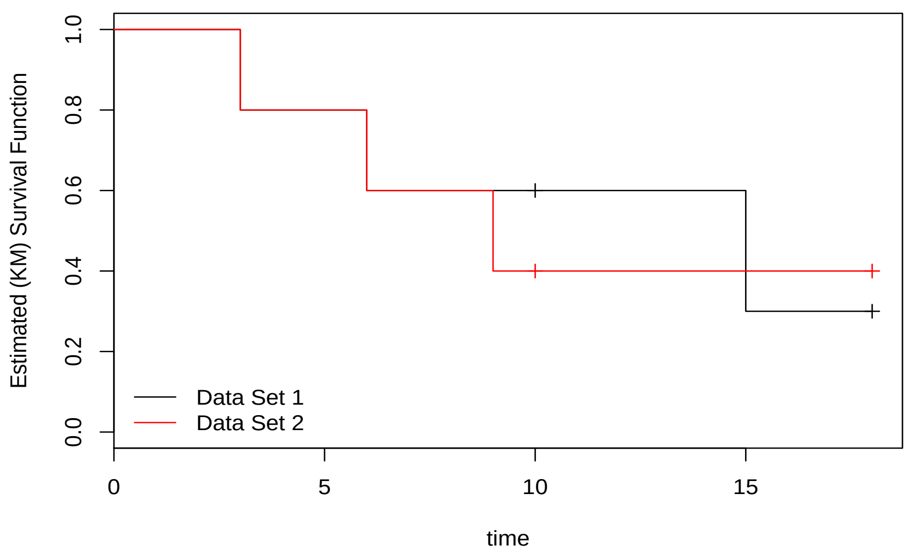
and the denominator is a summation of similar terms: $\sum_{j=1}^{n} S_1(t_0+dt)\Big[S_j(t_0) - S_j(t_0+dt)\Big]...S_n(t_0+dt)$. Thus the desired conditional probability simplifies to

$$\frac{\Big[S_k(t_0)-S_k(t_0+dt)\Big]/S_k(t_0+dt)}{\sum_{j=1}^{n}\Big[S_j(t_0)-S_j(t_0+dt)\Big]/S_j(t_0+dt)}\,.$$

Now divide both the numerator and the denominator by the factor dt, and let $dt \to 0$, we get

$$\frac{f_k(t_0)/S_k(t_0)}{\sum_{j=1}^{n} f_j(t_0)/S_j(t_0)} = \lambda_k(t_0)/(\lambda_1(t_0)+...+\lambda_n(t_0))\,.$$

5. The two survival estimates (KM) are plotted below. Note that the (estimated)



survival function for data set 1 is not always above that for data set 2, although it seems that data set 1 has longer survivals than data set 2. **This is because of the censoring at t=10**. By $t=9$, the two survival curves are identical (both take the value of $\frac{3}{5}$). At $t=9$, the 2nd curve has a jump (decrease) of size $\frac{3}{5} * \frac{\mathbf{1}}{\mathbf{3}} = \frac{1}{5}$, while the 1st curve does not decrease and remains $\frac{3}{5}$ until $t=15$, where the jump size becomes $\frac{3}{5} * \frac{\mathbf{1}}{\mathbf{2}} = \frac{3}{10} > \frac{1}{5}$.